200142 - EDPS - Partial Differential Equations

Coordinating unit: 200 - FME - School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics
981 - CRM - Mathematical Research Centre

Academic year: 2018
Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Teaching unit Compulsory)
ECTS credits: 7,5
Teaching languages: Catalan

Teaching staff
Coordinator: XAVIER CABRE VILAGUT
Others: Segon quadrimestre:
XAVIER CABRE VILAGUT - A, B
GYULA CSATO - B
TOMÁS SANZ PERELA - A

Degree competences to which the subject contributes

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking into account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

Generical:
5. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
6. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
7. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
8. CG-1. Show knowledge and proficiency in the use of mathematical language.
10. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
11. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
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**Teaching methodology**

(Section not available)

**Learning objectives of the subject**

(Section not available)

**Study load**

| Total learning time: 187h 30m | Hours large group: 45h 24.00% | Hours medium group: 0h 0.00% | Hours small group: 30h 16.00% | Guided activities: 0h 0.00% | Self study: 112h 30m 60.00% |
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## Content

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<th><strong>Introduction</strong></th>
<th><strong>Learning time:</strong> 29h 30m</th>
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<td>Self study : 15h 30m</td>
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**Description:**
1. Integration by parts; the heat equation from physical principles and the divergence theorem; boundary and initial conditions; well posed problems.
2. Examples of important PDEs and what they model. The linear transport equation.

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<th><strong>The diffusion or heat equation</strong></th>
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<td>Self study : 30h</td>
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**Description:**
3. The diffusion equation in bounded domains (separation of variables and Fourier series; energy method and uniqueness; maximum principle and uniqueness).
4. The diffusion equation in $\mathbb{R}^n$ (fundamental solution; the Dirac delta; convolution; existence and uniqueness theorem; regularity; non homogeneous equations and Duhamel principle).
5. The diffusion equation from random walk (random walk and propagation of errors; relation between caloric functions and probability densities and the Gaussian distribution).

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<th><strong>The Laplace and Poisson equations</strong></th>
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**Description:**
6. Properties of harmonic functions (examples; separation of variables and Poisson equation in a ball; mean value property, maximum principle and uniqueness; Harnack and Liouville properties; relation between harmonic functions, random walks, the discrete Laplacian and exit probabilities).
7. Fundamental solution and Green function (Newtonnian potential; Green function; reflection method: Green function for the half-space and the ball).
8. Dirichlet minimization principle and the energy method.
**First order equations**

**Learning time:** 25h  
Theory classes: 6h  
Practical classes: 4h  
Self study: 15h

**Description:**  
9. The linear transport equation (travelling waves, characteristics, stability).  
10. Quasilinear first order equations (examples: traffic dynamics, Burgers equation; method of the characteristics; the Riemann problem, shocks and entropy condition).

**The wave equation**

**Learning time:** 36h  
Theory classes: 7h 30m  
Practical classes: 6h  
Self study: 22h 30m

**Description:**  
11. Types of waves. Dispersion. The equation of the vibrating string (derivation; energy; separation of variables).  
12. The wave equation in R (d'Alembert formula; fundamental solution; non homogeneous equations; domains of dependence and of influence; propagation and reflection of waves). Classification of linear 2nd order PDEs: characteristics and canonical form.  
13. The wave equation in R^3 and R^2 (Kirchoff and Poisson formulae; Huygens principle).

**Qualification system**

First there will be a midterm exam (CP). At the end of the term there will be a final exam (F). The final subject mark will be the maximum between F and (0,3·CP+0,7·F).

An extra exam will take place on July for students that failed during the regular semester.

**Regulations for carrying out activities**

In the exams any kind of material, class notes or formularies will be forbidden. The midterm exam is not eliminatory.
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Bibliography

Basic:

Complementary: