200202 - TOPA - Algebraic Topology

Coordinating unit: 200 - FME - School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2018
Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Teaching unit Optional)
ECTS credits: 6
Teaching languages: Catalan

Teaching staff

Coordinator: JAUME AMOROS TORRENT
Others: Segon quadrimestre:
JAUME AMOROS TORRENT - A
GUILLEM BLANCO FERNÁNDEZ - A

Opening hours

Timetable: Anounced in Atenea.

Prior skills

Knowledge of the contents of the Topology course.
Understand the basic notions of geometry seen in the Affine and Euclidean Geometry course.
Calculate with Matlab / Octave.

Degree competences to which the subject contributes

Specific:
3. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
4. CE-4. Have the ability to use computational tools as an aid to mathematical processes.
5. Ability to solve problems from academic, technical, financial and social fields through mathematical methods.

Generical:
1. CB-4. Have the ability to communicate their conclusions, and the knowledge and rationale underpinning these to specialist and non-specialist audiences clearly and unambiguously.
2. To have developed those learning skills necessary to undertake further interdisciplinary studies with a high degree of autonomy in scientific disciplines in which Mathematics have a significant role.
6. CG-1. Show knowledge and proficiency in the use of mathematical language.
7. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
8. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
9. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
10. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.
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Transversal:

11. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.

12. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

Teaching methodology

Half of the time will be devoted to the presentation by the teacher of the contents of the subject and the other half will be devoted to the discussion and resolution of problems related to the contents.

Learning objectives of the subject

Familiarize the student with the computation of homology (groups and generators) in a wide range of topological spaces and versions of homology, by hand and by machine. Show geometrical applications of homology, both theoretical and applied to the recognition of images and to Topological Data Analysis.

Study load

<table>
<thead>
<tr>
<th>Total learning time: 150h</th>
<th>Hours large group: 30h</th>
<th>20.00%</th>
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<tbody>
<tr>
<td></td>
<td>Hours medium group: 0h</td>
<td>0.00%</td>
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<tr>
<td></td>
<td>Hours small group: 30h</td>
<td>20.00%</td>
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<tr>
<td></td>
<td>Guided activities: 0h</td>
<td>0.00%</td>
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<tr>
<td></td>
<td>Self study: 90h</td>
<td>60.00%</td>
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</tbody>
</table>

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## Content

<table>
<thead>
<tr>
<th>Course topics</th>
<th>Learning time: 120h</th>
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<tbody>
<tr>
<td></td>
<td>Theory classes: 30h</td>
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<tr>
<td></td>
<td>Practical classes: 30h</td>
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<td>Self study: 60h</td>
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### Description:

#### Lesson 1: Polyhedra
- Simplicial polyhedra, knot, star, morphisms.
- Simplicial homology, Orientation and sign, Interpretation of the groups in degrees zero and one.
- Theory 3h, problems 4h, personal work 5h

#### Lesson 2: Homological algebra
- Chain complexes, Morphisms, Homotopies, Homotopy invariance of homology.
- Snake lemma, homology long exact sequence.
- Simplicial Mayer-Vietoris and relative homology.
- Theory 6h, problems 8h, personal work 16h

#### Lesson 3: Singular and CW homology
- Singular homology, Mayer-Vietoris, cellular attachments.
- CW homology, Künneth.
- Theory 4h, problems 4h, personal work 6h

#### Lesson 4: Homology of varieties
- Smooth varieties and simplicial varieties, Local homology, homology of maximum degree.
- Intersection, Poincaré duality.
- Cell decomposition associated to a Morse function.
- Theory 5h, problems 6h, personal work 8h

#### Lesson 5: Topological data analysis
- Barcodes and persistent homology.
- Applications: image recognition, structure of natural images.
- Theory 8h, problems 4h, personal work 12h

### Expected dedication:
- Theory 26h
- Problems 26h
- Personal work 47h
- Computational assignment: 4h theory, 4h problems, 13h personal work.
The final qualification is the result of applying the following formula:

\[ N = \max \{ 0.5 \cdot E + 0.2 \cdot \text{PAR} + 0.3 \cdot \text{PRAC}, 0.7 \cdot E + 0.3 \cdot \text{PRAC} \} \]

where \( E \) will be the qualification obtained by the student in a global exam that will be done at the end of the semester, \( \text{PAR} \) will be the qualification obtained in a partial exam that will be done at the middle of the semester, and \( \text{PRAC} \) will be the qualification of a practice that the student will develop throughout the semester.

**Bibliography**

**Basic:**

**Complementary:**