220133 - Finite Elements in Structural Analysis

Learning objectives of the subject

The goal of this course is to introduce finite elements in the context of structural analysis. We will consider the basic theory of the method as utilized as a structural engineering tool. Different structural topologies will be considered, from truss elements (based on the matrix structural analysis) to shell elements; passing through beam, solid, and axisymmetric elements. The primary tool we will use to learn about the basis of the method will be programming some elements in the software MATLAB. The key steps of the computer implementation will be presented in sufficient detail so that the student can understand what goes on behind the scenes of a typical commercial code.

Study load

<table>
<thead>
<tr>
<th>Total learning time: 75h</th>
<th>Hours large group: 30h</th>
<th>40.00%</th>
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<tbody>
<tr>
<td></td>
<td>Hours medium group: 0h</td>
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<tr>
<td></td>
<td>Hours small group: 0h</td>
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<td>Guided activities: 0h</td>
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<td>Self study: 45h</td>
<td>60.00%</td>
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## Content

<table>
<thead>
<tr>
<th>Module</th>
<th>Direct approach for discrete systems</th>
<th>Learning time: 10h</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Theory classes: 4h</td>
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<td>Self study : 6h</td>
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**Description:**
- Description of a single bar element
- Displacement, strain, stress, constitutive relation
- Internal and external forces
- Equations for Assembly
- Boundary Conditions and Solution of the system
- 2D and 3D Truss, transformation law

<table>
<thead>
<tr>
<th>Module</th>
<th>One-Dimensional element (FEM 1D)</th>
<th>Learning time: 8h</th>
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<tbody>
<tr>
<td></td>
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<td>Theory classes: 3h 12m</td>
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<td>Self study : 4h 48m</td>
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</tbody>
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**Description:**
- One-dimensional elastic problem (strong form)
- The weak form in one dimension with arbitrary boundary conditions
- Equivalence between weak and strong forms
- Spatial discretization. Shape functions in one dimension.
- Elemental stiffness matrix. Assembling.
- Global stiffness matrix
- Development of discrete equation system
- Convergence by numerical experiments

<table>
<thead>
<tr>
<th>Module</th>
<th>Beam element</th>
<th>Learning time: 12h</th>
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<tbody>
<tr>
<td></td>
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<td>Theory classes: 5h</td>
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<td>Self study : 7h</td>
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</tbody>
</table>

**Description:**
- Review of general concepts
- Governing equations of the beam (strong form)
- Weak form. Integration by parts.
- Hermite polynomials for both the displacements and the derivatives of the displacements (rotations)
- Discrete equations
- Moments and shear forces diagrams
The final grade is based on three assignments, each contributing 33.3% of the final mark. Students whose grade happens to be below 50% will be allowed to present a complementary work in order to raise their grade up to 50% (but not higher). The contents of the complementary work will be at the discretion of the teacher, depending on the circumstances of each student. The deadline for delivering the complementary work will be 2 weeks after the end of the classes.

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**Module 4: Finite element in solids**

- Learning time: 20h
  - Theory classes: 8h
  - Self study: 12h

**Description:**
- General review. Displacements, strains, stresses, Hooke law, equilibrium equations, boundary conditions
- Virtual work principle (general case)
- Plane stress, Plane strain
- Triangular element, Quadratic element
- Numerical integration, Gauss quadrature in two dimensions

**Module 4: Solids of revolution**

- Learning time: 10h
  - Theory classes: 4h
  - Self study: 6h

**Description:**
- Elasticity relations for axial symmetry
- Axisymmetric solid element
- Discrete equations, Examples

**Module 5: Plate and shell elements**

- Learning time: 15h
  - Theory classes: 6h
  - Self study: 9h

**Description:**
- Reissner-Mindlin plate theory
- Plate-bending elements
- Doubly curved shells

**Qualification system**

The final grade is based on three assignments, each contributing 33.3% of the final mark. Students whose grade happens to be below 50% will be allowed to present a complementary work in order to raise their grade up to 50% (but not higher). The contents of the complementary work will be at the discretion of the teacher, depending on the circumstances of each student. The deadline for delivering the complementary work will be 2 weeks after the end of the classes.
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Bibliography

Basic:

