230080 - AL - Linear Algebra

Coordinating unit: 230 - ETSETB - Barcelona School of Telecommunications Engineering
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2019
Degree: BACHELOR'S DEGREE IN TELECOMMUNICATIONS TECHNOLOGIES AND SERVICES ENGINEERING (Syllabus 2015). (Teaching unit Compulsory)
ECTS credits: 6
Teaching languages: Catalan, Spanish

Teaching staff
Coordinator: Rafael Cubarsi

Degree competences to which the subject contributes

General:

2. ABILITY TO IDENTIFY, FORMULATE AND SOLVE ENGINEERING PROBLEMS Level 1. To identify the complexity of the problems presented in the subjects. To set out correctly the problem correctly from the statements suggested. To identify the possible options for its resolution. To choose an option, apply it and to identify the need to change it in case of fail. To provide tools and methods to test whether the solution is correct or at least consistent. To identify the role of creativity in science and technology

Transversal:

1. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

Teaching methodology

Application lectures
Expositive lectures
Personal work (non classroom)
Short-answer questions (Test)
Proves de resposta llarga (Examen Final)

Learning objectives of the subject

To introduce the basic concepts of linear algebra.

Learning outcome:
He/she expresses clearly the process of planning and solving exercises and problems that require the use of linear algebra.
He/she understands and masters the most useful methods to solve problems in the area of this subject.
He/she addresses numerical description and formulation of problems with descriptive description.
He/she makes use of more than one source and uses it in a complementary manner to observe the events described in the main text.
He/she identifies problems and models from open situations and explores alternative resolutions.
230080 - AL - Linear Algebra

### Study load

<table>
<thead>
<tr>
<th>Total learning time: 150h</th>
<th>Hours large group:</th>
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<tbody>
<tr>
<td></td>
<td>65h</td>
</tr>
<tr>
<td>Self study:</td>
<td>85h</td>
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<td>43.33%</td>
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<td>56.67%</td>
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# Content

| (ENG) Tema 1. Matrices and determinants. | Learning time: 23h  
Theory classes: 10h  
Self study : 13h |
<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>Description:</strong></td>
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<tr>
<td>Matrices and sub-matrices. Operations and properties.</td>
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<tr>
<td>Elementary transformations. Echelon forms.</td>
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<tr>
<td>Rank of a matrix. Inverse matrix.</td>
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<tr>
<td>Systems of linear equations. Discussion and resolution of systems.</td>
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<tr>
<td>Gaussian elimination. Gauss-Jordan elimination.</td>
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<tr>
<td>Determinants: definition and properties.</td>
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<tr>
<td>Calculation of determinants. Orthogonal matrices.</td>
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<tr>
<td>Minors and calculating the rank of a matrix by minors. Cramer's rule.</td>
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<tr>
<td>Traces and cofactors. Laplace's formula. Adjugate matrix.</td>
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| (ENG) Tema 2. Vectorial spaces. | Learning time: 28h 45m  
Theory classes: 12h 30m  
Self study : 16h 15m |
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<tbody>
<tr>
<td><strong>Description:</strong></td>
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<tr>
<td>Definition, properties, and examples.</td>
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<td>Linear independence.</td>
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<td>Generating system, basis and dimension.</td>
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<td>Components and change of basis.</td>
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<tr>
<td>Vector subspaces. Implicit equations.</td>
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<tr>
<td>Intersection, sum and direct sum. Grassmann formula.</td>
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<tr>
<td>The four subspaces associated to a matrix.</td>
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| (ENG) Tema 3. Euclidean space. | Learning time: 34h 30m  
Theory classes: 15h  
Self study : 19h 30m |
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<tbody>
<tr>
<td><strong>Description:</strong></td>
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<tr>
<td>Inner product, norm, and angle.</td>
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<tr>
<td>Cauchy-Schwarz and triangular inequalities, Pythagorean theorem.</td>
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<tr>
<td>Orthogonality. Orthonormal and orthonormal basis.</td>
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<tr>
<td>Change of basis. Positive definite matrices.</td>
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<tr>
<td>Orthogonal complement. Orthogonal projection and best approximation.</td>
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<tr>
<td>Best approximation for a linear system: least squares. Quadratic error.</td>
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<tr>
<td>Orthogonality of the fundamental subspaces.</td>
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<tr>
<td>Euclidean vector spaces of infinite dimension. Orthogonal polynomials and trigonometric functions.</td>
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<tr>
<td>Introduction to unitary space.</td>
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### (ENG) Tema 4. Linear transformations.

**Description:**
- Definition and properties. Associated matrix.
- Rank of a linear transformation.
- Kernel and image. Rank-nullity theorem.
- Injective and exhaustive transformations.
- Endomorphisms. Change of basis. Equivalent matrices.
- Invariant subspaces.

**Learning time:** 23h
- Theory classes: 10h
- Self study: 13h

### (ENG) Tema 5. Diagonalization of endomorphisms.

**Description:**
- Eigenvectors and eigenvalues.
- Characteristic polynomial and traces of an endomorphism.
- Eigenspaces, algebraic and geometric multiplicities.
- Diagonalization conditions.
- Complex eigenvalues of real matrices.
- Symmetric endomorphisms.
- Orthogonal basis of eigenvectors.
- Orthogonal diagonalization of symmetric matrices. Spectral theorem.
- Positive definite and semidefinite matrices.
- Singular value decomposition.

**Learning time:** 28h 45m
- Theory classes: 12h 30m
- Self study: 16h 15m

### (ENG) Tema 6. Complex matrices.

**Description:**
- Conjugate and hermitian matrices.
- Complex inner product.
- Unitary matrices.

**Learning time:** 11h 30m
- Theory classes: 5h
- Self study: 6h 30m
# Planning of activities

| (ENG) Test (Test) | Hours: 1h  
Theory classes: 1h |
|-------------------|---------------------|
| (ENG) Test (Test) | Hours: 1h  
Theory classes: 1h |
| (ENG) Exam (Final Exam) | Hours: 3h  
Theory classes: 3h |

# Qualification system

Two tests along course: 40%  
Final exam: 60%

# Bibliography

**Basic:**


**Complementary:**
