# 230461 - MM2 - Mathematical Methods 2

Coordinating unit: 230 - ETSETB - Barcelona School of Telecommunications Engineering
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2019
Degree: BACHELOR'S DEGREE IN ENGINEERING PHYSICS (Syllabus 2011). (Teaching unit Compulsory)
ECTS credits: 6 Teaching languages: Catalan

## Teaching staff

Coordinator: CARLES BATLLE ARNAU

Others: JAUME FRANCH BULLICH

## Degree competences to which the subject contributes

Specific:

2. Ability to select numerical and optimization methods suitable for solving physical and engineering problems. Ability to apply the knowledge of numerical algorithms and optimization.
1. Ability to solve math problems that may arise in engineering. Ability to apply knowledge about linear algebra, geometry, differential geometry, differential and integral calculus, ordinary and partial differential equations, probability and statistics.

Generical:

3. ABILITY TO IDENTIFY, FORMULATE, AND SOLVE PHYSICAL ENGINEERING PROBLEMS. Planning and solving physical engineering problems with initiative, making decisions and with creativity. Developing methods of analysis and problem solving in a systematic and creative way.

Transversal:

1. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.
2. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.

## Teaching methodology

Lectures (3 sessions per week): the instructor presents the fundamental concepts and some relevant proofs, intermingled with key examples and the solution of representative problems.
Recitations (2 sessions per week): the students review the fundamental concepts and solve some problems, under the guidance of the instructor.

## Learning objectives of the subject

To acquire the theoretic foundations and the techniques associated to the mathematical methods of physics, such as the calculus of variations, the function spaces, Fourier analysis, the linear differential operators, the separation of variables method, and the complex analysis.

## Study load

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| Total learning time: 150h | Hours large group: | 65h | 43.33% |
| | Self study: | 85h | 56.67% |

## Content

| Complex analysis | Learning time: 35h 30m<br>Theory classes: 9h<br>Practical classes: 5h<br>Guided activities: 0h 30m<br>Self study : 21h |
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Description:
Functions of a complex variable are introduced, and holomorphic functions are defined. Integration on paths in the complex plane is studied, and the main integral formulas are presented, as well as Laurent series expansions. The obtained results are applied to the computation of improper integrals.

Specific objectives:
- Functions of a complex variable. Holomorphic functions. Cauchy-Riemann equations.
- Integration on paths in the complex plane. Cauchy's theorem. Cauchy's integral formulas.
- Taylor and Laurent series. Singularities. Computation of residues.
- Theorem of the residues. Aplication to the computation of some improper integrals.

| Function spaces, Fourier series and Fourier transform | Learning time: 46h 30m<br>Theory classes: 11h<br>Practical classes: 7h<br>Guided activities: 0h 30m<br>Self study : 28h |
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Description:
Banach and Hilbert abstract spaces are introduced, and the special case of square-integrable functions is considered. In this space, families of orthogonal functions and their associated Fourier series are defined. In particular, the trigonometric and complex exponential families are presented, and the main convergence results for them are stated. Finally, the Fourier transform and some of its properties are studied.

Specific objectives:
- Banach and Hilbert spaces. Lp spaces. The space L2.
- Sets of orthogonal functions. Fourier series. trigonometric and exponential families.
- Convergence theorems: Bessel, Parseval, Dirichlet.
- Fourier transform. Inverse transform. Plancherel's theorem. Theorem of convolution.

| Linear differential operatos and Green functions | Learning time: 29h 30m<br>Theory classes: 9h<br>Practical classes: 6h<br>Guided activities: 0h 30m<br>Self study : 14h |
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Description:
The concept of adjoint operator associated to a differential operator is presented, and the boundary conditions that render an operator self-adjoint are discussed. Green functions and Sturm-Liouville problems are also defined.

Specific objectives:
- Linear differential operators.
- Adjoint operator. Hermitic and self-adjoint operators.
- Spectrum of a self-adjoint operator.
- Green functions. Initial value problems. Impulsive response.
- Properties of Green functions. Symmetry, non-homogeneous boundary conditions and eigenfunction expansion. Causality and analyticity.

| Partial differential equations | Learning time: 26h<br>Theory classes: 7h<br>Practical classes: 4h<br>Guided activities: 1h<br>Self study : 14h |
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Description:
The classification of second order linear PDEs is given, and some of the properties related to well-posedness are discussed in physical terms. Separation of variables techniques are presented.

Specific objectives:
- Classification of linear second order PDEs. Boundary and initial conditions.
- The method of characteristics for first-order EDP and for hyperbolic equations.
- The wave equation. D'alembert solution, causal Green function and separation of variables.
- The heat equation. Heat kernel and the method of images.
- Potential theory. Unicity of solutions, separation of variables, eigenfunctions and Green functions. Electrostatic potential.

## Qualification system

The grading of this course is based mainly on written examinations where the students will solve some problems, but some homework will also be taken into account.

Final grade: maximum of (PF, 65% PF + 25% PP + 10% PNP), where
PF final examination, covering all the material
PP partial examination, covering the first chapters
PNP homework

In order to assign the Honors ("Matrícula d'Honor") qualification, the ordering of grades computed with 65% PF + 25% PP + 10% PNP will be used.

## Regulations for carrying out activities

The written exams will be closed-book, but a specified number of sheets of notes will be allowed for each of them, as well as the computer support that the instructors deem necessary.

Homework norms will be provided at the beginning of the course.

## Bibliography

### Basic:

Stone, M.; Goldbart, P. Mathematics for physics: a guided tour for graduate students. Cambridge: Cambridge University Press, 2009. ISBN 978-0521854030.

Tang, K.T. Mathematical methods for engineers and scientists, vol.3: fourier analysis, partial differential equations and variational methods. Berlin: Springer, 2007. ISBN 9783540446958.

Beck, M.; Marchesi, G.; Pixton, D.; Sabalka, L. A first course in complex analysis [on line]. Versió 1.41. San Francisco, CA; Binghamton, NY: Els autors, 2002-2012 [Consultation: 25/02/2015]. Available on: <http://math.sfsu.edu/beck/papers/complex.pdf>.

### Complementary:

Batlle, C.; Fossas, E. Anàlisi real: apunts. Barcelona: Facultat de Matemàtiques i Estadística, UPC, Ahlens. D.L.: B-8830-2002, 2002.

Marsden, J.; Hoffman, M. Análisis clásico elemental. 2a ed. Buenos Aires: Addison-Wesley Iberoamericana, 1998. ISBN 978-0201653694.

Marsden, J.E.; Hoffman, M.J. Basic complex analysis. 3rd ed. New York: W.H. Freeman, 1999. ISBN 978-0716728771.

Stakgold, I.; Holst, M. Green's functions and boundary value problems. 3rd ed. Hoboken, New Jersey: John Wiley & Sons, 2010. ISBN 978-0-470-60970-5.