

## 230489 - RELG - General Relativity

Coordinating unit: 230 - ETSETB - Barcelona School of Telecommunications Engineering  
Teaching unit: 749 - MAT - Department of Mathematics  
748 - FIS - Department of Physics  
Academic year: 2019  
Degree: BACHELOR'S DEGREE IN ENGINEERING PHYSICS (Syllabus 2011). (Teaching unit Optional)  
ECTS credits: 6 Teaching languages: Spanish

### Teaching staff

Coordinator: NARCISO ROMÁN ROY  
Others: RAMÓN TORRES HERRERA

### Requirements

It is advisable to have studied the subject "Mechanics" (2A, Degree on Physics Engineering), or "Mathematical Models of Physics" (3B, Degree on Mathematics).

### Degree competences to which the subject contributes

Specific:

MAT1. Ability to solve math problems that may arise in engineering. Ability to apply knowledge about linear algebra, geometry, differential geometry, differential and integral calculus, ordinary and partial differential equations, probability and statistics.

3. Knowledge of the scientific method and its applications in physics and engineering. Ability to formulate hypotheses and make critical analysis of scientific problems in the field of physics and engineering. Ability to relate the physical reality with their mathematical models and vice versa.

FG2. Ability to solve basic problems in mechanics, elasticity, thermodynamics, fluids, waves, electromagnetism and modern physics, and its application in solving engineering problems.

General:

2. ABILITY TO IDENTIFY, FORMULATE, AND SOLVE PHYSICAL ENGINEERING PROBLEMS. Planning and solving physical engineering problems with initiative, making decisions and with creativity. Developing methods of analysis and problem solving in a systematic and creative way.

Transversal:

1. SELF-DIRECTED LEARNING - Level 3. Applying the knowledge gained in completing a task according to its relevance and importance. Deciding how to carry out a task, the amount of time to be devoted to it and the most suitable information sources.

### Teaching methodology

Lectures with interaction with students

### Learning objectives of the subject

The aims of this course are to provide sufficient training to students in order to advance the knowledge of the Theory of General Relativity (TRG). After introducing the basic mathematical tools and reviewing the Minkowskian formulation of the Theory of Special Relativity, we want to describe the conceptual progress that led Albert Einstein to formulate the TRG and we state this theory, its current relevance, the experimental tests that support it and the main contributions in the subject of the external field and stellar collapse, generation of black holes and the origin and evolution of the universe: cosmological solutions.



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### Study load

Total learning time: 150h	Hours large group:	65h	43.33%
	Self study:	85h	56.67%

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### Content

1. Complements of tensor algebra and differential geometry.

Learning time: 47h 20m

Theory classes: 10h  
Practical classes: 10h  
Self study : 27h 20m

Description:

Basic concepts on tensor algebra. Tensor fields. Operations. Vector fields, differential forms and tensor fields in  $R^n$ . Differentiable manifolds. Differentiable operators. Derivations. Covariant derivative. Christoffel symbols. Connections. Parallel transport. Geodesics. Equations. Metric tensor. Symmetries and Killing vectors. Vector fields and their invariant classification: Acceleration, Expansion, Shear, and Rotation. Torsion tensor of a connection. Levi-Civita connection. Curvature tensors (Riemann, Ricci), properties.

2. Review on Special Relativity: The Minkowskian formulation of the Special Relativity.

Learning time: 7h

Theory classes: 2h  
Practical classes: 1h  
Self study : 4h

Description:

Postulates of the Special Relativity. Minkowski's metrics and Minkowskian space-time. Inertial observers. Four-vectors. Light cone. Lorentzian geometry. Lorentz and Poincaré transformations and groups. Relativistic kinematics and relativistic dynamics. The electromagnetic tensor: Maxwell equations.

Four-interval. Light cone and causality. Four-velocity and four-acceleration: transformation laws. Equations of relativistic dynamics: Conservation of four-momentum and relativistic Newton's law. Invariants and Minkowski metric. The space-time of the Special Relativity. The electromagnetic tensor: Maxwell equations.

3. Principles of General Relativity.

Learning time: 3h 30m

Theory classes: 1h  
Practical classes: 1h  
Self study : 1h 30m

Description:

Newtonian gravitation. Inertial and gravitational mass. The principle of equivalence, consequences. Non-Euclidean geometries. Postulates of General Relativity.

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<p>4. The equations of General Relativity.</p>	<p>Learning time: 11h 50m Theory classes: 3h 20m Practical classes: 1h 40m Self study : 6h 50m</p>
<p>Description: The stress-energy-momentum tensor. Conservation of the stress-energy-momentum tensor. The Einstein tensor. Properties. Einstein field equations and geodesic equation. Consequences of Einstein equations. Tidal effects. The cosmological constant. General relativity from a variational principle: the Hilbert-Einstein Lagrangian. Phenomenological aspects.</p>	
<p>5. Kinematics in General Relativity.</p>	<p>Learning time: 6h 30m Theory classes: 2h Practical classes: 2h Self study : 2h 30m</p>
<p>Description: Time-like geodesic congruencies. Distances and time intervals in general relativity. Three-dimensional metric tensor. The synchronization criteria of clocks in General Relativity. Locally-inertial reference frame. Gravitational red-shift. A covariant generalization of Doppler effect and gravitational red-shift.</p>	
<p>6. The Schwarzschild solution. Geodesics in Schwarzschild geometry.</p>	<p>Learning time: 16h 20m Theory classes: 4h Practical classes: 3h Self study : 9h 20m</p>
<p>Description: Solutions with spherical symmetry. Static solutions. Asymptotically flat solutions. The Schwarzschild spacetime. Properties and the Birkhoff theorem. Singularities of the curvature (essential, intrinsic or real) and removable singularities (singularities of the coordinates). Lagrangian method to obtain the equation of timelike (or null) geodesics. Symmetries and conserved quantities. Bound orbits. Stable and unstable circular orbits. Radial geodesics.</p>	

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7. Experimental tests of General Relativity.	Learning time: 7h 10m Theory classes: 2h Practical classes: 1h Self study : 4h 10m
Description: Advance of the perihelion of Mercury. Deflection of light rays. Gravitational Red-Shift. The electromagnetic waves time delay.	
8. Black holes.	Learning time: 11h 50m Theory classes: 3h Practical classes: 2h Self study : 6h 50m
Description: Eddington-Finkelstein coordinates. Event horizon. Black holes. A classical argument. Tidal forces in a black hole. Observational evidence for black holes.	
9. Maximal extension and conformal compactification.	Learning time: 11h 40m Theory classes: 3h Practical classes: 2h Self study : 6h 40m
Description: Maximal analytic extensions. The Kruskal solution. Penrose diagram of Minkowski's space-time. Penrose diagram of the Schwarzschild-Kruskal space-time. Black holes versus white holes. Trapped surfaces closed. Gravitational collapse of a star. Creation of black holes.	
10. Radiation modeling and collapse: Vaidya solution	Learning time: 11h Theory classes: 2h Practical classes: 2h Self study : 7h
Description: External spacetime of a star that emits or receives radiation: Vaidya metric. Flux of radiation. Stress-energy-momentum tensor. Black holes and radiation. Penrose diagrams of Vaidya metric. Radiating collapse of a spherically symmetric space-time: Matching conditions. Dominant energy conditions in a radiative collapse.	

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11. Relativistic Cosmology. Cosmological models.	Learning time: 15h 50m Theory classes: 4h Practical classes: 3h Self study : 8h 50m
Description: Olbers' Paradox. Hubble's Law. The cosmological principle. Weyl's postulate. Friedmann equations. Relativistic cosmology. The geometry of 3-spaces of constant curvature. The flat space models. Friedmann-Lemaître-Robertson-Walker models in a flat case. Big-Bang. Conformal structure of a "flat" Friedmann-Lemaître-Robertson-Walker model.	

### Qualification system

Two partial exams (P1 and P2). Final exam (F).  
 Final qualification: The best of  $(P1 + P2) / 2$  and F.  
 Reevaluation exam.

### Bibliography

Basic:

- D'Inverno, R. *Introducing Einstein's relativity*. Oxford: Clarendon Press . Oxford University Press, 1992. ISBN 0198596863.
- Misner, C.W.; Thorne, K.S.; Wheeler, J.A. *Gravitation*. San Francisco: W. H. Freeman and Company, 1973. ISBN 0716703440.
- Landau, L.D. *The classical theory of fields* [on line]. 4th rev English ed. Oxford: Pergamon, 1980 [Consultation: 10/10/2018]. Available on: <<https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?docID=1675199>>. ISBN 9781483293288.
- Carroll, S.M. *Spacetime and geometry: an introduction to general relativity*. New Intern. ed. Essex: Pearson, 2014. ISBN 9781292026633.
- Poisson, E. *A relativist's toolkit: the mathematics of black-hole mechanics*. Cambridge, UK ; New York: Cambridge University Press, 2004. ISBN 0521830915, 9780511606601.