Course guide
2500012 - GECCVECTEQ - Vector Calculus and Differential Equation

Unit in charge: Barcelona School of Civil Engineering
Teaching unit: 751 - DECA - Department of Civil and Environmental Engineering.
Degree: BACHELOR'S DEGREE IN CIVIL ENGINEERING (Syllabus 2020). (Compulsory subject).
Academic year: 2021 ECTS Credits: 6.0 Languages: Spanish

LECTURER
Coordinating lecturer: ENRIQUE BENDITO PEREZ
Others: ENRIQUE BENDITO PEREZ

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES
Specific:
14392. Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge about: linear algebra; geometry; differential geometry; differential and integral calculation; differential equations and partial derivatives; numerical methods; numerical algorithmic; Statistics and optimization. (Basic training module)

TEACHING METHODOLOGY
The course consists of 2 hours per week of classroom activity (large size group) and 2 hours weekly with half the students (medium size group).

The 2 hours in the large size groups are devoted to theoretical lectures, in which the teacher presents the basic concepts and topics of the subject, shows examples and solves exercises.

The 2 hours in the medium size groups is devoted to solving practical problems with greater interaction with the students. The objective of these practical exercises is to consolidate the general and specific learning objectives.

Support material in the form of a detailed teaching plan is provided using the virtual campus ATENEA: content, program of learning and assessment activities conducted and literature.
LEARNING OBJECTIVES OF THE SUBJECT


1. Ability to relate differential equations in ordinary with engineering problems.
2. Ability to program simple solutions through basic software and to obtain numerical solutions.
3. Ability to develop solutions to these problems under simple conditions that allow an analysis of these solutions, including a parametric study.


Introduction and general features of PDEs (Definition. Examples from mathematical physics. Linear PDE (superposition, homogeneous). Basic tools: divergence theorem).


Classification of second order equations

Review of three important examples of PDEs. Motivating example. A general framework. Domain dependence and general features of PDEs
The weak form

Derivation of the weak form of Laplace's equation and the heat equation.

STUDY LOAD

<table>
<thead>
<tr>
<th>Type</th>
<th>Hours</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours medium group</td>
<td>30,0</td>
<td>20.00</td>
</tr>
<tr>
<td>Hours large group</td>
<td>30,0</td>
<td>20.00</td>
</tr>
<tr>
<td>Self study</td>
<td>84,0</td>
<td>56.00</td>
</tr>
<tr>
<td>Guided activities</td>
<td>6,0</td>
<td>4.00</td>
</tr>
</tbody>
</table>

**Total learning time:** 150 h
CONTENTS

CHANGE OF VARIABLES

Description:
Evidence the practical utility of the realization of the Theorem of the Inverse Function.
Describe the most interesting curvilinear coordinates in two- and three-dimensional Euclidean space.
Consideration of vector spaces, of the same dimension of the coordinated object, in each one of its points.
Construction of the bases of vector spaces as tangent vectors to coordinate curves.

Specific objectives:
Understand the concept of variable change respecting the properties of regularity.
Learn to make the most common variable changes.
Describe the appropriate space to consider vector fields restricted to coordinated objects.
Understand the Jacobian of curvilinear coordinates as the matrix of the change of variable for vector fields.

Full-or-part-time: 12h
Theory classes: 2h
Practical classes: 3h
Self study: 7h

PARAMETERIZED CURVES

Description:
Change of parameter and tangent to curves. Qualitative and quantitative analysis of parameterized curves.
Qualitative knowledge of regular parameterized curves. Learn to obtain the curves as well as their main properties, both in the case of regular and arc parameterizations.

Specific objectives:
Know the different kinematic concepts associated with the trajectory of a point in geometric terms and the properties of trajectories.
Construct the minimum information necessary to distinguish the parameterized curves except movement of rigid solid.

Full-or-part-time: 9h 36m
Theory classes: 2h
Practical classes: 2h
Self study: 5h 36m

PARAMETERIZED SURFACES

Description:
Understand the structure of surfaces and control their regularity.
Determine the surfaces and their appearance by the way they are described.

Specific objectives:
Distinguish why an application is a surface and evaluate when it has a tangent plane
Know the ruled, revolution and level surfaces.

Full-or-part-time: 7h 11m
Theory classes: 1h
Practical classes: 2h
Self study: 4h 11m
MANIFOLDS WITH BOUNDARY

Description:
Opens of dimension 2 and 3 with boundary and the First Fundamental Form. Consider parameterizations of closed intervals and pseudo-intervals, characterizing the induced orientation at the boundary of the parameterization.

Specific objectives:
Give a common language to curves, surfaces and open two and three-dimensional and introduce the ability to measure. Know the description of a wide variety of objects as a finite union of pseudo-intervals or as the parameterization of them.

Full-or-part-time: 7h 11m
Theory classes: 2h
Practical classes: 1h
Self study : 4h 11m

INTEGRATION

Description:
Consideration of the measure elements and the coordinate elements. Obtaining the Stokes Theorem at parameterized intervals, by applying the Barrow rule for each variable. Obtaining the Divergence, Stokes-Ampère and Green Theorems in the plane, as an elementary application of the general Stokes theorem. Solving exercises in which the most characteristic resources appear that facilitate the resolution of a wide range of problems.

Specific objectives:
Understand the scope of the process of integration of functions and the correspondence between the realization of the integration inside the objects or in the boundary of them. Understand the relationship between divergence, flow and circulation of vector fields in three- and two-dimensional objects and their boundaries. Learn to solve integration problems on complex objects but which are described in pieces using simple expressions that make it easier to perform calculations on them.

Full-or-part-time: 28h 47m
Theory classes: 5h
Practical classes: 7h
Self study : 16h 47m

EXAMINATION 1

Full-or-part-time: 7h 11m
Laboratory classes: 3h
Self study : 4h 11m
VARIATION OF INTEGRALS

Description:
Know the variation of integrals with respect to time, when both the sub-integral function and the integration domain depends on him.

Obtaining conservation laws as an application of integration results and basic mechanical concepts in continuous media.

Specific objectives:
Using elementary techniques of integration and applying classical theorems, connect with fundamental elements in the description of continuous media.
Analyze fundamental physical laws from a simplified point of view, emphasizing vector calculus techniques.

Full-or-part-time: 9h 36m
Theory classes: 4h
Self study: 5h 36m

CALCULATION OF VARIATIONS

Description:
Formulation of the problem, necessary boundary conditions, Euler and Euler-Lagrange equation and natural boundary conditions. Go through problems that arise through the necessary condition of extreme, detecting the characteristics of the types of equations obtained, self-adjoint problems, of various variables, of eigenvalues, with natural or forced boundary conditions. Construction and knowledge of the main equations of mathematical physics. Explain the correspondence between the differential equations of mathematical physics and the variational principles such as the minimum potential energy, the principle of minimum action, electrostatic balance or elastic balance and pose the problems of vibration of strings, beams or membranes.

Specific objectives:
The aim is to have a systematic way of formulating problems, which covers the most relevant part of the problems that arise in the context of the differential equations of both one and several variables. It is intended to come into contact with a number of problems that broaden the horizon of action, as is the case with the equations from the minimization of the elastic deformation energy. It is intended to provide tools to build the problems of mathematical physics. Access to some of the great problems of mathematical physics, from a physical-mechanical perspective that motivates the consideration of the equations to be solved and the methods to do so.

Full-or-part-time: 14h 23m
Theory classes: 4h
Practical classes: 2h
Self study: 8h 23m

QUADRATIC FUNCTIONAL

Description:
Classical classification criteria and prototypes of elliptic, parabolic, and hyperbolic equations are proposed.

Specific objectives:
Present families of problems that can be addressed: heat equations, wave equation, Laplace equation, eigenfunctions, domain types and boundary and initial value conditions.

Full-or-part-time: 7h 11m
Theory classes: 3h
Self study: 4h 11m
SEPARATION OF VARIABLES

Description:
The Fourier method is introduced from one-dimensional diffusion problems, leading to any problem that fits this method of resolution. This will allow us to solve many previously posed problems by representing solutions on orthonormal bases.

Specific objectives:
Having a tool for obtaining effective analytical solutions to problems of interest in many applied areas and also understanding its framework of application, on the other hand allows to assess the need for other techniques to address problems that are beyond the reach of the method.

Full-or-part-time: 16h 48m
Practical classes: 7h
Self study : 9h 48m

APPROXIMATE SOLUTIONS

Description:
The weak way is to write the differential equations in an integral way and with less regularity requirements and they are easy to systematize.
To propose a methodology of control of the coefficients of the problems to treat the approximate resolution in spaces of finite dimension. Solution of evolution problems.

Specific objectives:
Consider alternative methods to the separation of variables, for the solution of problems of the maximum interest and expose the rudiments so that the weak form is the route of application of methods of approach for many problems.
Access the basic rudiments so that, throughout their studies, they can know the methods of numerical solution of many problems of interest including the problems of evolution.

Full-or-part-time: 16h 48m
Theory classes: 3h
Practical classes: 4h
Self study : 9h 48m

EXAM 2

Full-or-part-time: 7h 11m
Laboratory classes: 3h
Self study : 4h 11m

GRADING SYSTEM

The mark of the course is obtained from the ratings of continuous assessment and their corresponding laboratories and/or classroom computers.

Continuous assessment consist in several activities, both individually and in group, of additive and training characteristics, carried out during the year (both in and out of the classroom).

The teachings of the laboratory grade is the average in such activities.

The evaluation tests consist of a part with questions about concepts associated with the learning objectives of the course with regard to knowledge or understanding, and a part with a set of application exercises.
BIBLIOGRAPHY

Basic:

Complementary: