Degree competences to which the subject contributes

Specific:
3051. Ability to deal with and solve advanced mathematical engineering problems, from the posing of the problem to its formulation and implementation in a computer program.
3055. Ability to formulate, programme and apply analytical and numerical calculation models to design, planning and management. Ability to interpret the results provided by the models in the civil engineering context.

Transversal:
592. EFFICIENT ORAL AND WRITTEN COMMUNICATION - Level 2. Using strategies for preparing and giving oral presentations. Writing texts and documents whose content is coherent, well structured and free of spelling and grammatical errors.
596. TEAMWORK - Level 1. Working in a team and making positive contributions once the aims and group and individual responsibilities have been defined. Reaching joint decisions on the strategy to be followed.
599. EFFECTIVE USE OF INFORMATION RESOURCES - Level 3. Planning and using the information necessary for an academic assignment (a final thesis, for example) based on a critical appraisal of the information resources used.
602. SELF-DIRECTED LEARNING - Level 3. Applying the knowledge gained in completing a task according to its relevance and importance. Deciding how to carry out a task, the amount of time to be devoted to it and the most suitable information sources.
584. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.

Teaching methodology

Taught module delivery: fifteen weeks of teaching, coursework and self-study. Apart from the 6 hours per week in the classroom, self-study must last an average of 9 hours per week.

At least a half of the classroom hours are devoted to work in small groups (computer laboratory, problem solving, etc.)
250133 - MODNUMER - Numerical Modelling

Students will learn to formulate and program analytical and numerical methods of calculation and apply them to project planning and management. They will also learn to interpret the results generated by the models in an engineering context.

Upon completion of the course, students will have acquired the ability to: 1. Use numerical analysis software to carry out a sensitivity analysis of a problem in which an ordinary differential equation is solved. 2. Use a partial differential equation to solve a boundary value problem in continuous media, finding a numerical solution, on the basis of the partial derivatives, by means of the finite difference method or the finite element method. 3. Use numerical techniques to solve modelling problems in an engineering context.

History of numerical models and their application to engineering projects; Numerical modelling in engineering; Number storage, algorithms and error analysis; Numerical methods for determining the root of a function; Solving systems of equations by means of direct numerical methods and basic iterative methods; Using numerical methods to solve nonlinear systems of equations; Eigenvalue problems; Function approximation; Numerical quadrature; Solving ordinary differential equations; Solving partial differential equations: finite difference methods and finite element methods

Intended Learning Outcomes:
1.- To demonstrate a knowledge and understanding of: the fundamentals of the behaviour and numerical approximation of differential equations; functional approximation; truncation error and solution error; consistency, stability and convergence; direct and iterative solution of linear systems of equations and eigenvalue problems.

2.- To demonstrate an ability to (thinking skills): understand and formulate basic numerical procedures and solve illustrative problems; identify the proper methods for the corresponding problem.

3.- To demonstrate an ability to (practical skills): understand practical implications of behaviour of numerical methods and solutions; logically formulate numerical methods for solution by computer with a programming language (Matlab or Octave).

4.- To demonstrate an ability to (key skills): study independently; use library resources; use a personal computer for basic programming; effectively take notes and manage working time.

<table>
<thead>
<tr>
<th>Study load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total learning time: 225h</td>
</tr>
<tr>
<td>Hours large group: 31h</td>
</tr>
<tr>
<td>Hours medium group: 11h</td>
</tr>
<tr>
<td>Hours small group: 48h</td>
</tr>
<tr>
<td>Guided activities: 9h</td>
</tr>
<tr>
<td>Self study: 126h</td>
</tr>
</tbody>
</table>
250133 - MODNUMER - Numerical Modelling

Content

1.- Basics on modeling, programming and error

Description:
Introduction to MATLAB programming
Basics on modeling, programming and error

Specific objectives:
Basics on MATLAB or OCTAVE to study independently.
Brief introduction to the module.
Basics on programming.
Knowledge on integers and real number representation in a computer.
Ability to understand the concept and definitions of error (absolute, rounding, truncation, significant digits) and basics on propagation.

Learning time: 43h 12m
Theory classes: 2h
Laboratory classes: 16h
Self study: 25h 12m

2.- Introduction to FEM

Description:
Practical example to determine the weak form 1D+2D
Obtaining the weak form: for the mechanical problem (principle of virtual work) and the Laplace problem (weighted residuals)
Sectional interpolation
Discretization of the weak form
Calculation of integrals
Elementary matrices and assembly
Tutorial: solving an equilibrium problem with FEM

Specific objectives:
To demonstrate knowledge and understanding of the detailed procedure, using weighted residuals, to obtain the weak form from the strong one for a boundary value problem.
To know applications of the FEM in engineering.
To be able to determine the weak form for elliptic problems with Dirichlet, Neumann or Robin boundary conditions.
To demonstrate knowledge and understanding of the various numerical aspects of MEF: discretization / approximation, integration, assembly, system solving, ...

Learning time: 19h 12m
Theory classes: 2h
Laboratory classes: 6h
Self study: 11h 12m
### 3.- Systems of linear equations

<table>
<thead>
<tr>
<th>Learning time: 28h 47m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory classes: 6h</td>
</tr>
<tr>
<td>Practical classes: 2h</td>
</tr>
<tr>
<td>Laboratory classes: 4h</td>
</tr>
<tr>
<td>Self study : 16h 47m</td>
</tr>
</tbody>
</table>

### Description:
- Classification and definitions.
- Factorization methods: Crout and Cholesky
- Iterative methods, conjugate gradient (analog with minimization problem, maximum descent, conjugate gradient algorithm, properties)
- Practical problems of systems of equations: storage, condition number, preconditioning
- Tutorial: solving systems by direct and iterative methods

### Specific objectives:
- To demonstrate knowledge and understanding of:
  - the classification of the methods of solving systems of linear equations,
  - the range of applicability of each method and its computational advantages and disadvantages,
  - the detailed analysis of the conjugate gradient method and how to implement it properly.

- To demonstrate knowledge and understanding of the basic practical aspects on: renumbering, condition number, preconditioners, significant digits, convergence criteria.
- To demonstrate an ability to implement the methods of resolution presented.
- To demonstrate an ability to study the practical influence the condition number, preconditioners ...
4.- Roots of functions and nonlinear systems

**Description:**
Basics of iterative methods: consistency, linear convergence, superlinear or order p, convergence rate, asymptotic factor, order.
Methods: Newton, secant, Whittaker.

Practical problems of convergence analysis and influence of rounding errors.

**Specific objectives:**
To demonstrate knowledge and understanding of:
iterative schemes and their inherent differences with finite operations methods,
the properties, advantages and disadvantages of standard iterative schemes,
the most appropriate method.

To demonstrate an ability to analyze, represent and interpret the results of iterative methods.
To demonstrate knowledge and understanding of the basic extensions to systems of equations.

<table>
<thead>
<tr>
<th>Test #1</th>
<th>Learning time: 4h 48m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Laboratory classes: 2h</td>
</tr>
<tr>
<td></td>
<td>Self study : 2h 48m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Learning time: 14h 23m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory classes: 3h</td>
</tr>
<tr>
<td>Practical classes: 1h</td>
</tr>
<tr>
<td>Laboratory classes: 2h</td>
</tr>
<tr>
<td>Self study : 8h 23m</td>
</tr>
</tbody>
</table>
# 5.- Interpolation and approximation

<table>
<thead>
<tr>
<th>Learning time:</th>
<th>14h 23m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory classes:</td>
<td>4h</td>
</tr>
<tr>
<td>Laboratory classes:</td>
<td>2h</td>
</tr>
<tr>
<td>Self study :</td>
<td>8h 23m</td>
</tr>
</tbody>
</table>

## Description:
- General approach: type and criteria for functional approximation
- Polynomial interpolation
- Least squares
- Splines

Tutorial: using function approximation for decision making.

## Specific objectives:
To demonstrate knowledge and understanding of:
- the criteria and types of functional approximation and their advantages and disadvantages,
- Lagrange interpolation and its error and an ability to use it,
- the least squares problem, namely to deduce the normal equations and understand the approximation orthogonality,
- splines.

To demonstrate an ability to use and code some intrinsic functions to approximate functions.
To demonstrate an ability to use several functional approximation techniques to analyze a data set and discuss the results.
6.- Numerical integration

**Description:**
General approach, eg with trapezoidal rule
Definition of order of a quadrature
Quadrature classification
Newton-Cotes formulas
Gauss quadrature
Composite formulae
An example of numerical integration: FEM
Tutorial: numerical integration

**Specific objectives:**
To demonstrate knowledge and understanding of:
the concept of numerical integration and ability to define a quadrature if the integration points are given,
the classification of the different quadratures,
the basis of the Newton-Cotes and Gaussian quadratures, an ability to use them choosing the correct one in
terms of accuracy and computational cost,
composite quadratures and their advantages and disadvantages.
To demonstrate an ability to apply all the concepts of numerical integration to the FEM.
To demonstrate an ability to implement an algorithm for numerical integration.
To demonstrate an ability to implement an algorithm for composite formulae.
To demonstrate an ability to analyze and discuss convergence of the following quadratures:
Newton-Cotes and Gauss-Legendre as they increase the number of integration points,
composite formulae as they increase the number of intervals.

---

**Test #2**

**Learning time:** 4h 48m
Laboratory classes: 2h
Self study : 2h 48m
## 7.- Modeling with ODEs

<table>
<thead>
<tr>
<th>Learning time: 19h 12m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory classes: 3h</td>
</tr>
<tr>
<td>Practical classes: 1h</td>
</tr>
<tr>
<td>Laboratory classes: 4h</td>
</tr>
<tr>
<td>Self study : 11h 12m</td>
</tr>
</tbody>
</table>

### Description:
- Problems with ODEs: backwater curves, cantilever with large displacements, evolution of pollutants.
- General approach: reduction to first order, initial value (IVP), boundary value (BVP) or eigenvalue problem, existence and uniqueness theorem.
- Methods based on the approximation of the derivative: Euler, backward Euler.
- Truncation error, consistency, local and global error, order, absolute stability.
- Single step methods (Runge-Kutta) methods: second and fourth order.
- Prediction-correction methods.

### Tutorial:
- Modelling in engineering with ODEs.
- Numerical solution of ODEs.

### Specific objectives:
- To demonstrate an ability to state and analyze various engineering problems that require solving ODEs.
- To demonstrate knowledge and understanding of:
  - well posed IVP,
  - definition and classification of ODEs (for any order and dimension) and ability to write it as $\frac{dy}{dx} = f(x, y)$,
  - classification into an IVP or BVP,
  - concepts of convergence, order of convergence and absolute stability.
- Runge-Kutta methods: general form and algorithm (Butcher table), basic characteristics (one-step methods, stability increases with order), differences between explicit, implicit and semi-implicit methods.
- To demonstrate an ability to model an engineering problem as a system of ODEs.
- To demonstrate an ability to use a library for the numerical solution of ODEs.
8.- Modeling with PDEs

**Description:**
Equilibrium (solid mechanics, soils ...), evolution (structural dynamics, heat, consolidation, traffic, pollutants ...) and eigenvalue problems (structural vibration, acoustics ...).

Mathematical and numerical classification of PDEs.
Finite element method for elliptic problems: boundary conditions, organization of the calculations, concepts of accuracy and numerical efficiency.
Parabolic problems: stability and time accuracy

(Hyperbolic) Dynamic problems: mass, stiffness and damping matrix; Newmark method
Tutorial: stability

**Specific objectives:**
To demonstrate an ability to analyze, represent and interpret various engineering problems that require solving PDEs.
To demonstrate knowledge and understanding of:
- identification and classification of second order PDEs, from a mathematical and physical point of view,
- the meaning of the boundary conditions,
- the dimensionless form of initial or boundary value problems (in particular "heat"),
- the first and second order difference operators,
- the basics of the FEM: weak form, boundary conditions, integration, types of matrices, algorithm,
- the concepts of precision in FEM (elliptic) and how to identify problems and propose solutions.

Understand the concepts of (conditional or unconditional) stability and time accuracy, and the difference between explicit and implicit methods.

To demonstrate knowledge and understanding of:
- the fundamental features of dynamic problems. Time-integration with Newmark's method
To demonstrate an ability to program Newmark's method and analyze its behavior

---

**Test #3**

**Learning time:** 4h 48m
- Laboratory classes: 2h
- Self study: 2h 48m
250133 - MODNUMER - Numerical Modelling

<table>
<thead>
<tr>
<th>Course assignment</th>
<th>Learning time: 4h 48m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Laboratory classes: 2h</td>
</tr>
<tr>
<td></td>
<td>Self study : 2h 48m</td>
</tr>
</tbody>
</table>

**Description:**
Presentation and discussion of modelling assignment

**Specific objectives:**
To demonstrate an ability to present, analyze and critically discuss a complete problem with numerical modeling

**Qualification system**

1. The module is graded with the following elements:
   - Class work (CW), to be carried out either individually or in small teams.
   - Two tests (T1 and T2), which are strictly individual.

2. Class work (CW) refers, among others, to:
   - Matlab coding in the computer room.
   - Problem solving in the classroom.
   - Participation in class.

3. Tests T1 and T2 will cover all the material taught from the beginning of the module.

4. Academic dishonesty (including, among others, plagiarism and falsification of results) will be severely punished, in accordance with current academic regulations: any such act will imply a final mark of 0 in the module.

5. The final mark for the module is obtained as

   \[
   \text{Mark} = \text{CW}^{(4/10)} \times \text{T}^{(6/10)} \quad \text{with} \quad \text{T} = (\text{T1} + \text{T2})/2
   \]

6. Criteria for re-evaluation qualification and eligibility: Students that failed the ordinary evaluation and have regularly attended all evaluation elements will have the opportunity of carrying out a re-evaluation test during the period specified in the academic calendar. Students who have already passed the test or were qualified as non-attending will not be admitted to the re-evaluation test. The maximum mark for the re-evaluation exam will be five over ten (5.0). The non-attendance of a student to the re-evaluation test, in the date specified will not grant access to further re-evaluation tests. Students unable to attend any of the continuous assessment tests due to certifiable force majeure will be ensured extraordinary evaluation periods.

   These tests must be authorized by the corresponding Head of Studies, at the request of the professor responsible for the course, and will be carried out within the corresponding academic period.

**Regulations for carrying out activities**

Will be discussed in class.
250133 - MODNUMER - Numerical Modelling

Bibliography

Basic:


Complementary:


