300010 - CAL-T - Calculus

Degree competences to which the subject contributes

On completing the Calculus course, students should be able to:

- Sketch a graph of the main one variable elementary functions.
- Solve simple inequations.
- Operate with logarithms and exponentials.

Prior skills

Ability in arithmetic calculations and in simplifying algebraic expressions.
Knowledge of the concept of function and of the graphic representation of a function.
Ability to think in abstract terms.

Degree competences to which the subject contributes

Specific:
1. CE 1 TELECOM. Students will acquire the ability to solve mathematical problems for engineering. An aptitude for applying knowledge of linear algebra, geometry, differential geometry, differential and integral calculus, differential equations and partial differential equations, numerical methods, numerical algorithms, statistics and optimisation. (CIN/352/2009, BOE 20.2.2009)

Transversal:
2. EFFICIENT ORAL AND WRITTEN COMMUNICATION - Level 1. Planning oral communication, answering questions properly and writing straightforward texts that are spelt correctly and are grammatically coherent.
3. TEAMWORK - Level 1. Working in a team and making positive contributions once the aims and group and individual responsibilities have been defined. Reaching joint decisions on the strategy to be followed.
4. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

Teaching methodology

In the theory lectures we will introduce the fundamental concepts of the subject, and will present the basic exercise and problem solving techniques.

In the problems sessions, exercises and problems proposed a priori by the lecturer and autonomously prepared by the students will be discussed and solved.

Learning objectives of the subject

On completing the Calculus course, students should be able to:

- Sketch a graph of the main one variable elementary functions.
- Solve simple inequations.
- Operate with logarithms and exponentials.
- Interpret the concept of derivative of a function in physical and geometric terms.
- Calculate derivatives of functions.
- Approach and solve optimisation problems involving one variable functions.
- Compute Taylor polynomials and approximate its Lagrange remainder.
- Understand the geometrical meaning of gradient of a function of two variables.
- Identify conics and quadrics from their equations.
- Use several techniques for calculating primitives.
- Calculate the area of plane regions and the volume of some solids in space.
- Operate with binomial and exponential forms of complex numbers.
- Polynomial factoring, over the fields of real numbers and complex numbers.

### Study load

<table>
<thead>
<tr>
<th>Total learning time: 150h</th>
<th>Hours large group: 39h</th>
<th>26.00%</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Hours medium group: 13h</td>
<td>8.67%</td>
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<tr>
<td></td>
<td>Hours small group: 0h</td>
<td>0.00%</td>
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<tr>
<td></td>
<td>Guided activities: 14h</td>
<td>9.33%</td>
</tr>
<tr>
<td></td>
<td>Self study: 84h</td>
<td>56.00%</td>
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</tbody>
</table>
# Content

## Content 1: Equations and graphics

**Learning time:** 26h 12m  
Theory classes: 6h 36m  
Practical classes: 2h 12m  
Guided activities: 2h 24m  
Self study: 15h

**Description:**
1.1 Straight line: basic concepts, equations, properties.  
1.2 Conics: introduction, parabola, circle, ellipse, hyperbola, identification of conics according to its equation.  
1.3 Elementary functions: introduction, polynomial and rational functions, exponential and logarithmic functions, absolute value function, trigonometric functions and their inverses.

**Related activities:**
Test C1, Mid term exam and Final term exam.

## Content 2: Derivation of functions of one variable

**Learning time:** 36h 12m  
Theory classes: 9h 36m  
Practical classes: 3h 12m  
Guided activities: 3h 24m  
Self study: 20h

**Description:**
2.1 Concept of derivative: definition, geometric interpretation, the derivative function.  
2.2 Calculation of derivatives: properties of the derivative, high-order derivatives, the chain rule, derivative of the inverse function, implicit derivation, logarithmic derivation.  
2.3 Tangent and normal straight lines.  
2.4 The criterion of L'Hôpital.  
2.5 Extrema of a function: absolute and relative extrema, increase and decrease of functions, determining relative extrema using the first derivative, concavity and convexity, determining relative extrema using the second derivative, criterion of the n-th derivative, absolute extrema on a closed interval, absolute extrema on a non closed interval.  
2.6 Optimization problems.  
2.7 Taylor polynomial.

**Related activities:**
Directed activity AD1, Mid term exam and Final term exam.
### Content 3: Integration of functions of one variable

**Description:**
- 3.1 Indefinite integral: primitive of a function, concept of indefinite integral, geometric interpretation, differential of a function, first properties of the indefinite integral.
- 3.2 Calculation of primitives: immediate integrals, linearity of the indefinite integral, almost immediate integrals, integration by parts, integration of rational functions, integration by change of variable, integration of trigonometric functions, integration of irrational functions.
- 3.3 Definite integral: definition, properties of the definite integral, the Fundamental Theorem of Calculus, Barrow’s rule, change of variable in the definite integral.
- 3.4 Applications of the definite integral: calculation of areas of planar figures, calculation of volumes of solids of revolution.
- 3.5 Improper integrals.

**Related activities:**
- Test C2 and Final term exam.

**Learning time:** 36h 12m
- Theory classes: 9h 36m
- Practical classes: 3h 12m
- Guided activities: 3h 24m
- Self study: 20h

### Content 4: Functions of two variables

**Description:**
- 4.1 Functions of two variables: definition, contour lines, contour map, $\mathbb{R}^3$ surfaces, sections, implicit equation of a surface, quadrics.
- 4.2 Derivation in two variables: slope of a curve on a surface, directional derivatives, partial derivatives, slope of a surface, tangent plane, normal line, tangent plane and normal line of an implicitly given surface, gradient of a function of two variables, properties of the gradient.

**Related activities:**
- Final term exam.

**Learning time:** 25h 12m
- Theory classes: 6h 36m
- Practical classes: 2h 12m
- Guided activities: 2h 24m
- Self study: 14h
Content 5: Complex numbers

Learning time: 26h 12m
- Theory classes: 6h 36m
- Practical classes: 2h 12m
- Guided activities: 2h 24m
- Self study: 15h

Description:
5.1 The imaginary unit.
5.2 Binomic form of a complex number: definition, operations in binomic form.
5.3 The complex plane.
5.4 Exponential form of a complex number: definition, operations in exponential form, trigonometric formulas.
5.5 n-th roots of a complex number.
5.6 Fundamental theorem of Algebra. Factoring polynomials

Related activities:
Test C1, Mid term exam and Final term exam.
# Planning of activities

<table>
<thead>
<tr>
<th>Activity 1: Test C1</th>
<th>Hours: 1h</th>
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<tbody>
<tr>
<td>Description:</td>
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<tr>
<td>Test of Contents 5 and 1.</td>
<td>1h</td>
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<tr>
<td>Theory classes:</td>
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<table>
<thead>
<tr>
<th>Activity 2: Test C2</th>
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<tbody>
<tr>
<td>Description:</td>
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<tr>
<td>Test of Content 3.</td>
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<tr>
<td>Theory classes:</td>
<td>1h</td>
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<thead>
<tr>
<th>Activity 3: Directed activity AD1</th>
<th>Hours: 1h 30m</th>
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<tbody>
<tr>
<td>Description:</td>
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<tr>
<td>Directed activity on using Taylor's polynomial to approximate functions.</td>
<td>1h 30m</td>
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<tr>
<td>Guided activities:</td>
<td>1h 30m</td>
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<table>
<thead>
<tr>
<th>Activity 4: Mid term exam</th>
<th>Hours: 1h 30m</th>
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<tbody>
<tr>
<td>Description:</td>
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<tr>
<td>Exam of Contents 1 and 2.</td>
<td>1h 30m</td>
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<tr>
<td>Guided activities:</td>
<td>1h 30m</td>
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<table>
<thead>
<tr>
<th>Activity 5: Final term exam</th>
<th>Hours: 2h</th>
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<tbody>
<tr>
<td>Description:</td>
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<tr>
<td>Test of Contents 1, 2, 3, 4 and 5.</td>
<td>2h</td>
</tr>
<tr>
<td>Guided activities:</td>
<td>2h</td>
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</tbody>
</table>
Qualification system

Test C1: 15 %
Test C2: 15 %
Mid term exam: 25 %
Final term exam: 40 %
Directed activity AD1 : 5%

Note: The final term exam allows to remediate the mid term exam mark. In case it is effectively remediated, the mark of the mid term exam will be replaced by that of the final term exam, which will therefore weight a 65% of the final grade. However, in order to ensure a real continuous assessment only students who have attended all exams and tests (passed or failed) will be eligible to remediate the mid term exam.

Regulations for carrying out activities

Tests are made during lecture sessions and are previously announced in ATENEA. The mid and final semester exams are made according to the EETAC's schedule.

Tests and exams are made individually. The use of books, notes, calculator, software and mobile phone is not allowed.

Bibliography

Basic:

Complementary:

Others resources:
- Course schedule with syllabus.
- Initial knowledge material.
- List of exercises (and answers) of the course.
- Sample models of exams and tests of previous courses.

All of them are available in ATENEA.