300018 - ALA - Linear Algebra and Applications

Coordinating unit: 300 - EETAC - Castelldefels School of Telecommunications and Aerospace Engineering
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2018
Degree: BACHELOR'S DEGREE IN TELECOMMUNICATIONS SYSTEMS ENGINEERING (Syllabus 2009).
(Teaching unit Compulsory)
BACHELOR'S DEGREE IN NETWORK ENGINEERING (Syllabus 2009). (Teaching unit Compulsory)
ECTS credits: 6
Teaching languages: Catalan, Spanish

Teaching staff

Coordinator: Definit a la infoweb de l'assignatura.
Others: Definit a la infoweb de l'assignatura.

Prior skills

The ability to carry out arithmetic calculations and simplify algebraic expressions.
Familiarity with the concept of a function and the graphic representation of a function.
The ability to apply differential and integral calculus of one or more variables.
Familiarity with the Laplace transform.
Capacity for abstract thinking.

Requirements

Upper secondary school mathematics.
Students must have taken or be taking Calculus and Mathematics for Telecommunications.

Degree competences to which the subject contributes

Specific:
1. CE 1 TELECOM. Students will acquire the ability to solve mathematical problems for engineering. An aptitude for applying knowledge of linear algebra, geometry, differential geometry, differential and integral calculus, differential equations and partial differential equations, numerical methods, numerical algorithms, statistics and optimisation. (CIN/352/2009, BOE 20.2.2009)

Transversal:
2. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.
3. EFFICIENT ORAL AND WRITTEN COMMUNICATION - Level 1. Planning oral communication, answering questions properly and writing straightforward texts that are spelt correctly and are grammatically coherent.
4. TEAMWORK - Level 1. Working in a team and making positive contributions once the aims and group and individual responsibilities have been defined. Reaching joint decisions on the strategy to be followed.
Teaching methodology

Two sessions lasting 90 minutes each per week in which theoretical concepts are dealt and example problems are solved. These sessions combine expository and participatory (cooperative learning) models.

One session lasting an hour per week is devoted to solving problems using a software tool such as Wiris or Maple. It also introduces the basic concepts that students will work on in the directed activities.

One hour per week of directed activities in which students present, analyse and solve practical cases analytically and numerically (using the software mentioned above). Problem-/project-based learning methods are used to carry out the directed activities. There are two types of directed activity sessions. In the first type, students will work individually on a guide downloaded from Atenea that contains a background study. Students will also hand in a questionnaire before carrying out the laboratory assignment. The second type of directed activity, a continuation of the first, will be carried out in the laboratory. Students will be expected to use mathematical software to solve a linear algebra problem following written guidelines. Directed activities will be conducted under the guidance of the lecturer. The reduced size of the groups allows the shortcomings of the previous activity to be detected and enhances students’ understanding of course topics. The lecturer can also provide individual guidance. This is also beneficial for the more able students. The lecturer’s proximity ensures continuous assessment.

Self-directed learning hours must be devoted to studying the course content and doing the exercises set by the lecturer.

Learning objectives of the subject

On completion of Linear Algebra and Applications, students will:

- Understand basic algebraic structures, Boolean algebra and modular arithmetic.
- Be able to solve linear systems of equations.
- Be able to carry out operations with matrices.
- Be familiar with the properties of vector spaces.
- Understand linear applications, changes of basis and matrix diagonalisation.
- Be able to carry out operations with scalar products and manipulate bases. Be familiar with orthonormalisation.
- Be able to geometrically interpret and solve the most common first-order differential equations, linear differential equations of order n and systems of first-order linear differential equations with constant coefficients. Be able to find particular solutions.
- Be able to use a computer tool (Wiris, Matlab or equivalent) to solve linear algebra problems.

Study load

<table>
<thead>
<tr>
<th>Total learning time: 150h</th>
<th>Hours large group:</th>
<th>Hours medium group:</th>
<th>Hours small group:</th>
<th>Guided activities:</th>
<th>Self study:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>39h</td>
<td>0h</td>
<td>13h</td>
<td>14h</td>
<td>84h</td>
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<tr>
<td></td>
<td>26.00%</td>
<td>0.00%</td>
<td>8.67%</td>
<td>9.33%</td>
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</table>
### Algebraic structures

**Description:**
1.1 Binary operations and properties. Algebraic structures (semigroups, groups, rings, fields).
1.3 Reticles and Boolean algebra. Logical circuits.
1.4 Polynomials.

**Related activities:**
Test 1, level test 1, directed activities 1, 2, 3 and 4

**Learning time:** 36h
- Theory classes: 9h
- Laboratory classes: 3h
- Guided activities: 4h
- Self study: 20h

### Linear systems of equations, matrices and determinants

**Description:**
2.2 Determinants.
2.3 Linear equation systems. Discussion and solution of systems. Cramer’s rule. The superposition principle.

**Related activities:**
Level test 1; Directed activities 5 and 6

**Learning time:** 23h
- Theory classes: 6h
- Laboratory classes: 2h
- Guided activities: 2h
- Self study: 13h
### Vector spaces

<table>
<thead>
<tr>
<th>Description:</th>
<th>Learning time: 23h</th>
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<tbody>
<tr>
<td>3.1 Vector spaces and subspaces. Subspace generated by a set: linear combinations. Linear dependence and independence. Generator systems.</td>
<td>Theory classes: 6h</td>
</tr>
<tr>
<td>3.2 Bases. Dimension. Coordinates of a vector basis. Change of basis.</td>
<td>Laboratory classes: 2h</td>
</tr>
<tr>
<td>3.3 Operations with subspaces: direct sum.</td>
<td>Guided activities: 2h</td>
</tr>
<tr>
<td><strong>Related activities:</strong></td>
<td>Self study: 13h</td>
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<tr>
<td>Test 2; Level test 2; Directed activities 7 and 8</td>
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</tbody>
</table>

### Linear operators and diagonalisation

<table>
<thead>
<tr>
<th>Description:</th>
<th>Learning time: 34h</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 Definitions and properties. Kernel and image. Matrix of a linear application. Change of basis in linear applications.</td>
<td>Theory classes: 9h</td>
</tr>
<tr>
<td>4.2 Diagonalisable endomorphisms and matrices. Eigenvectors and eigenvalues. Characteristic polynomial.</td>
<td>Laboratory classes: 3h</td>
</tr>
<tr>
<td>4.3 Diagonalisation. Cayley-Hamilton theorem. First decomposition theorem.</td>
<td>Guided activities: 3h</td>
</tr>
<tr>
<td>4.4 Scalar product. Orthogonal and orthonormal bases. Orthogonal projection.</td>
<td>Self study: 19h</td>
</tr>
<tr>
<td><strong>Related activities:</strong></td>
<td></td>
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<tr>
<td>Test 3; Level test 2; Directed activities 9, 10, 11 and 12</td>
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</table>
Differential equations

<table>
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<tr>
<th>Learning time: 34h</th>
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<tbody>
<tr>
<td>Theory classes: 9h</td>
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<tr>
<td>Laboratory classes: 3h</td>
</tr>
<tr>
<td>Guided activities: 3h</td>
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<tr>
<td>Self study: 19h</td>
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Description:
5.1 First-order differential equations Definition. Separable, linear and homogeneous equations. Exact differential equations.
5.2 Higher-order linear differential equations with constant coefficients. Test method for obtaining a particular solution in the inhomogeneous case.
5.3 Linear differential systems of equations with constant coefficients. Substitution method. Homogenous and inhomogeneous systems. Application of the Laplace transform.

Related activities:
Test 4; Level test 3; Directed activities 13 and 14
### Planning of activities

| TEST 1 | Hours: 10h 45m  
|        | Theory classes: 0h 45m  
|        | Self study: 10h |

**Description:**  
Individual test. Two or three exercises similar to the ones in the list of problems worked on in class.

**Support materials:**  
Lecture notes and list of the problems available on the digital campus

**Descriptions of the assignments due and their relation to the assessment:**  
Test.  
Assessment: See section on tests.

**Specific objectives:**  
Binary operations and properties. Euclidean algorithms. Modular arithmetic. Boolean operations.

| TEST 3 | Hours: 10h 45m  
|        | Theory classes: 0h 45m  
|        | Self study: 10h |

**Description:**  
Individual test. Two or three exercises similar to the ones in the list of problems worked on in class.

**Support materials:**  
Lecture notes and list of the problems available on the digital campus

**Descriptions of the assignments due and their relation to the assessment:**  
Test.  
Assessment: See section on tests.

**Specific objectives:**  
For students to work with linear operators and diagonalisation and calculate eigenvalues and eigenvectors.

| TEST 4 | Hours: 10h 45m  
|        | Theory classes: 0h 45m  
|        | Self study: 10h |

**Description:**  
Individual test. Two or three exercises similar to the ones in the list of problems worked on in class.

**Support materials:**  
Lecture notes and list of the problems available on the digital campus

**Descriptions of the assignments due and their relation to the assessment:**  
Test.  
Assessment: See section on tests.

**Specific objectives:**  
For students to work with linear operators and diagonalisation and calculate eigenvalues and eigenvectors.
### DIRECTED ACTIVITIES 1 AND 2 (INTRODUCTION TO MATHEMATICAL SOFTWARE)

**Description:**
Introduction to free and University-licensed software and how it might be used in the course. The software should allow students to operate with matrices and solve linear systems of equations. (e.g.

**Support materials:**
Materials AD2 and AD3 (available on the digital campus)

**Descriptions of the assignments due and their relation to the assessment:**
- Assignment 1: Completed questionnaire.
- Assignment 2: Methodological design and completed assignment.
- Assessment: See the section on group assignments.

**Specific objectives:**
To assess students' familiarity with basic sets of instructions in linear algebra software.

<table>
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<tr>
<th>Hours</th>
<th>Guided activities: 2h</th>
<th>Self study: 2h</th>
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### DIRECTED ACTIVITIES 3 AND 4 (MODULAR ARITHMETIC AND RSA ENCRYPTION)

**Description:**
The background study describes the mathematics of RSA encryption. Students will complete and hand in a questionnaire at the beginning of the second directed activity, which is carried out in the laboratory under the guidance of the lecturer. The directed activity consists in designing the methodology and solving a practical RSA public key encryption exercise using the concepts and methods contained in the background study.

**Support materials:**
Materials AD3 and AD4 (available on Atenea)

**Descriptions of the assignments due and their relation to the assessment:**
- Assignment 3: Completed questionnaire.
- Assignment 4: Methodological design and completed assignment.
- Assessment: See section on group assignments

**Specific objectives:**
For students to apply modular arithmetic to RSA encryption

<table>
<thead>
<tr>
<th>Hours</th>
<th>Guided activities: 2h</th>
<th>Self study: 2h</th>
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### DIRECTED ACTIVITIES 13 AND 14 (DIFFERENTIAL EQUATIONS)

**Description:**

**Support materials:**

**Descriptions of the assignments due and their relation to the assessment:**

**Specific objectives:**

<table>
<thead>
<tr>
<th>Hours</th>
<th>Guided activities: 2h</th>
<th>Self study: 2h</th>
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</table>
**Description:**
The background study involves revising systems of equations and their applications to predator-prey models. Students hand in a questionnaire at the beginning of the second directed activity, which is carried out in the laboratory under the guidance of the lecturer and involves designing a methodology and solving problems such as an adapted predator-prey race model. (sections 11.20 and 11.21 of Google's 2005 annual report)

**Support materials:**
Materials AD13 and AD14 (available on Atenea)

**Descriptions of the assignments due and their relation to the assessment:**
Assignment 13: Completed questionnaire.
Assignment 14: Methodological design and completed assignment.
Assessment: See section on group assignments

**Specific objectives:**
Applying numerical methods for solving differential equations to solve prey-predator models (for example propagation of agents in networks, etc.)

### DIRECTED ACTIVITIES 5 AND 6 (MATRICES AND NETWORKS)

**Description:**
The background study describes the relationship between matrices and networks (adjacency matrix). Students hand in a questionnaire at the beginning of the second directed activity, which is carried out in the laboratory under the guidance of the lecturer. Students use real and simulated network data to carry out practical network property calculations for the matrix involving hubs and distances and operations. They also consider matrix products and column sums.

**Support materials:**
Materials AD5 and AD6 (available on Atenea)

**Descriptions of the assignments due and their relation to the assessment:**
Assignment 5: Completed questionnaire.
Assignment 6: Methodological design and completed assignment.
Assessment: See section on group assignments

**Specific objectives:**
For students to apply matrix operations to the study of networks.

### DIRECTED ACTIVITIES 7 AND 8 (VECTOR SPACES AND ERROR-CORRECTING CODES)

**Description:**
The background study describes the relationship between vector spaces and matrices and linear error-correcting codes. Students hand in a questionnaire at the beginning of the second activity, which is carried out in the laboratory under the guidance of the lecturer and involves designing the methodology and solving practical cases of coding-decoding.
Support materials:
Materials AD7 and AD8 (available on Atenea)

Descriptions of the assignments due and their relation to the assessment:
Assignment 7: Completed questionnaire.
Assignment 8: Methodological design and completed assignment.
Assessment: See section on group assignments

Specific objectives:
For students to apply vector spaces and matrix operations to the study of linear error-correcting codes.

DIRECTED ACTIVITIES 9 AND 10
(DIAGONALISATION)

Description:
The background study describes methods for finding eigenvalues and eigenvectors. Students hand in a questionnaire at the beginning of the second activity, which is carried out in the laboratory under the guidance of the lecturer. The activity involves designing a methodology and carrying out eigenvalue and eigenvector calculations (e.g. Google's PageRank algorithm and sections 11.15 and 11.17 of its 2005 annual report).

Support materials:
Materials AD9 and AD10 (available on Atenea)

Descriptions of the assignments due and their relation to the assessment:
Assignment 9: Completed questionnaire.
Assignment 10: Methodological design and completed assignment.
Assessment: See section on group assignments

Specific objectives:
Diagonalisation. Power method. Relation to matrix power. Applications (e.g. PageRank and agent status).

DIRECTED ACTIVITIES 11 AND 12 (LINEAR OPERATORS)

Description:
The background study shows the uses and scope of affine transformations (plane linear transformations) in fractal image compression. Students hand in a questionnaire at the beginning of the second activity, which is carried out in the laboratory under the guidance of the lecturer and involves solving specific cases of fractal compression (section 11.14 of Google's 2005 annual report).

Support materials:
Materials AD11 and AD12 (available on Atenea)

Descriptions of the assignments due and their relation to the assessment:
Assignment 11: Completed questionnaire.
Assignment 12: Methodological design and completed assignment.
Assessment: See section on group assignments
Specific objectives:  
Apply affine transformations for fractal image compression.

<table>
<thead>
<tr>
<th>Qualification system</th>
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<tbody>
<tr>
<td>Number</td>
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<tr>
<td>Examinations</td>
</tr>
<tr>
<td>Tests</td>
</tr>
<tr>
<td>Level tests</td>
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<tr>
<td>Directed activities subject to assessment (*)</td>
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</tbody>
</table>

- Students must hand in all the assignments, two at a time. If they do not hand in an assignment, they will be given a mark of 0. Four assignments per group will be assessed.

Regulations for carrying out activities

Tests are held during theory-based lectures and last approximately 20 minutes.
The first examination is sat halfway through the semester (the specific week is set by the School).
The second examination is sat a week after classes have ended.
The examinations last 90 minutes. The same examination in set for all the groups.
Three individual level tests are taken on Atenea.
Four of the directed activity assignments will be assessed.
Bibliography

Basic:


Complementary:


Others resources:

Material available on the digital campus (Atenea):

1) Specific materials for the cooperative learning (Jigsaw) sessions in three sections (roles):
2) Lecture notes.
3) Sets of problems.
4) Background study and questionnaire for the first group of directed activities and script and questionnaire for the second group.