Course guides
320048 - E - Elasticity

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Unit in charge: Terrassa School of Industrial, Aerospace and Audiovisual Engineering
Teaching unit: 712 - EM - Department of Mechanical Engineering.

Degree: BACHELOR'S DEGREE IN MECHANICAL ENGINEERING (Syllabus 2009). (Compulsory subject).

Academic year: 2020 **ECTS Credits:** 6.0 **Languages:** Catalan

LECTURER

Coordinating lecturer: Javier Alvarez del Castillo

Others: Albert Catalan

PRIOR SKILLS

Students will be expected to have passed the subject Electrical Systems (third semester).

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
5. MEC: Students will acquire the skills and knowledge necessary to apply the fundamentals of the elasticity and strength of materials to the behaviour of real solids.

Transversal:
1. SELF-DIRECTED LEARNING - Level 2: Completing set tasks based on the guidelines set by lecturers. Devoting the time needed to complete each task, including personal contributions and expanding on the recommended information sources.
2. SUSTAINABILITY AND SOCIAL COMMITMENT - Level 2. Applying sustainability criteria and professional codes of conduct in the design and assessment of technological solutions.
3. EFFICIENT ORAL AND WRITTEN COMMUNICATION - Level 2. Using strategies for preparing and giving oral presentations. Writing texts and documents whose content is coherent, well structured and free of spelling and grammatical errors.
4. ENTREPRENEURSHIP AND INNOVATION - Level 2. Taking initiatives that give rise to opportunities and to new products and solutions, doing so with a vision of process implementation and market understanding, and involving others in projects that have to be carried out.

TEACHING METHODOLOGY

· Face-to-face lecture sessions.
· Face-to-face practical work sessions.
· Independent learning and exercises.
· Cooperative learning.
In the lectures, the lecturer will introduce the theoretical fundamentals of the subject, concepts, methods and results, which will be illustrated with relevant examples to facilitate their understanding. Practical class work will be covered in four types of sessions:
a) Sessions in which the lecturer will provide students with guidelines to analyse data for solving problems by applying methods, concepts and theoretical results.
b) Sessions in which students give presentations of group work.
c) Sessions in which cooperative learning takes place.
d) Sessions in which students are tested.
Students will be expected to study in their own time so that they are familiar with concepts and are able to solve the exercises set, whether manually or with the help of a computer.

LEARNING OBJECTIVES OF THE SUBJECT

A) Conceptual goals
1. To give students all of the basic training in mechanical engineering they need to understand the subjects on the degree course and subsequently apply the knowledge acquired in their professional lives.
2. To give students the training and knowledge required to design simple #resistors# and determine their precise size so that they are able to withstand the forces to which they will be subjected. The components designed should meet safety standards and be able to prevent failures due to the lack of resistance or instability and with stress levels compatible with their functionality. That is; To sum up, this means dimensioning the parts and based on the load to which they will be subjected, calculating the displacement of prismatic parts and resolving statically indeterminate systems.
3. To give solid grounding that will enable students who take the specialisation to follow courses at a higher level that cover the study of machine components and more complex structures in general, and that will enable them to learn new techniques.
4. To introduce students to a number of specific case studies related to the various fields they can go on explore on the basis of taking a specialisation in mechanics.

B) Procedural goals
1. To promote the skills and abilities that will help students make progress in their engineering studies and that will serve them in their professional lives, such as:
- The ability to identify and solve problems.
- The ability to think creatively.
- The ability to take decisions and assess the impact of all the factors involved.
- The ability to communicate effectively.
- The ability to analyse, synthesise and assess data.
- The ability to compile information.
- The ability to work both in a team and individually.
- The ability to embrace an entrepreneurial spirit.
- The ability to take a systemic and holistic point of view.
2. To promote the skills necessary to take on the design of systems and components, processes, installations and products that meet specific needs.
3. To promote the skills necessary to design and conduct experiments, as well as to interpret their results.
4. To ensure students are proficient users of the modern engineering techniques necessary to work in a professional context.
5. To teach students how to think as engineers by making them see the importance of an initial hypothesis in supporting the validity of a given solution, the fact that there are many possible solutions to a particular problem and that all aspects of a problem must be researched (including the initial premise), from the admissible margins of error and confidence intervals to the research of an optimum solution.
6. To develop students' learning abilities and to motivate them so that they are continuously updating and enhancing their skills.
C) Attitudinal goals
1. To make students aware of their ethical and professional responsibilities, and of the need for them to become active participants in the development of their local communities and society as a whole.
2. To ensure students are able to assess the impact of engineering solutions in a global, environmental and social context.
3. To encourage students to adopt values of responsibility, honesty, active participation, curiosity, the desire to learn, critical thinking, persistence, tenacity, drive and a commitment to social wellbeing.
4. To engage with students in such as way that as professionals they uphold and spread the expertise and prestige of the engineering profession.

STUDY LOAD

Type	Hours	Percentage
Hours medium group	30,0	20.00
Self study	90,0	60.00
Hours large group	30,0	20.00

Total learning time: 150 h

CONTENTS

TOPIC 1: INTRODUCTION TO ELASTICITY AND STRENGTH OF MATERIALS

Description:
1.1 Introduction to elasticity.
1.2 Rational mechanics, strength of materials and theory of elasticity.
1.3 Rigid and deformable bodies.
1.4 Equilibrium of deformable bodies. Fundamental postulates.
1.5 Superposition principle.

Specific objectives:
In this topic, we explore the concept of matter by relating it to rational mechanics, and in doing so highlight the continuity and progress of scientific knowledge. We also cover the fundamental postulates on which the equilibrium of deformable systems is based.

Full-or-part-time: 5h
Theory classes: 2h
Self study : 3h

TOPIC 2: STATE OF STRESS

Description:
2.1 Concept of stress at a point.
2.2 Stress vector. Intrinsic components.
2.3 State of stress in the neighbourhood of a point.
2.4 Stress tensor.
2.5 Equilibrium equations at internal and external points.
2.6 Calculation of the intrinsic components of a stress vector.
2.7 Stresses and principal axes.
2.8 Invariants of the stress tensor.
2.9 Octahedral stresses.
2.10 Decomposition of stress tensors: spherical and deviator tensors.
2.11 Lamé's ellipsoid.
2.12 Two-dimensional representation of a stress tensor. Mohr's circles. Maximum shear stress.
2.13 Applied exercises.

Specific objectives:
This topic explores the concept of stress state. We begin by covering the concept of stress at a point, the stress vector associated with the orientation of a plane in the neighbourhood of a point, and the stress tensor as a general expression of the state of stress in the neighbourhood of a point, from which any stress vector associated with any particular orientation can be found and defined in terms of the stress vectors associated with the three Cartesian coordinate planes. We then cover the relationship between the stress tensor and external forces, derived from the required equilibrium between the internal and external points of the body. We discuss the calculation of the intrinsic components of stress vectors (normal and shear), of stresses and principal axes, of invariants of the stress tensor, of octahedral stresses, and of the decomposition of spherical stress tensors and stress deviator tensors. We then cover the two-dimensional (Mohr's circle) and three-dimensional (Lamé's ellipsoid) representations of the state of stress in the neighbourhood of a point, as opposed to the mathematical representation of stress (stress tensor), highlighting the large amount of information conveyed by these representations.
The following exercises will be carried out as part of this topic: reflection on the concept of equilibrium between the internal and external points of a body in equilibrium that involve calculating the state of stress; applied exercises involving the calculation of the Cartesian and intrinsic components of a stress vector for a given plane orientation in the neighbourhood of a point, whether by taking the stress tensor in said neighbourhood or, failing that, by using external forces in cases with simple geometries and loads; reflection on the principle of reciprocity; the calculation of stresses and principal axes; and various applications of Mohr's circles to three-dimensional stress states.

Full-or-part-time: 25h
Theory classes: 5h
Practical classes: 5h
Self study : 15h

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TOPIC 3: STATE OF STRAIN

Description:
3.1 Infinitesimal transformation in the neighbourhood of a point. Statement of the transformation. Assumptions made.
3.2 Components of the displacement: translation, rotation and pure deformation.
3.3 Strain tensor.
3.4 Unitary strain vector in any direction. Unit elongation. Slip.
3.5 Angular deformation. Angular distortion.
3.6 Physical interpretation of the components of the strain tensor.
3.7 Principal strains and directions.
3.8 Invariants of the strain tensor.
3.9 Decomposition of strain tensors: spherical and deviator tensors.
3.10 Octahedral strain.
3.11 Strain quadrics.
3.12 Two-dimensional representation of a strain tensor.
3.13 Compatibility conditions for strain tensors.
3.14 Calculation of displacement.
3.15 Applied exercises.

Specific objectives:
In this topic, we explore the concept of state of strain in parallel to the state of stress, highlighting the fact that both are defined in terms of their respective tensors, both of which are second-order symmetric tensors, such that the mathematical treatment is the same for both states. We then focus on the physical interpretation of the components of the two tensors. We finish by studying the compatibility conditions for strain tensor components, with a focus on their physical significance and the calculation of displacement.
It is essential that students gain a thorough understanding of the knowledge imparted in these first three topics, both from a mathematical point of view and, especially, in terms of the physical significance of each of the concepts covered. Only with this knowledge will they be able to follow the rest of the subject.
The following exercises will be carried out as part of this topic: applied exercises involving the calculation of strain state given the vector of displacement in the neighbourhood of a point, as well as reflection on the associated concepts of translation, rotation and deformation; the calculation of displacements using strain tensors and applying compatibility equations; exercises involving the calculation of pure strain vectors, longitudinal unit elongation, slip and angular deformations in a given direction in the neighbourhood of a point; and the calculation of principal strains and directions and applications of Mohr's circle.

Full-or-part-time: 25h
Theory classes: 5h
Practical classes: 5h
Self study : 15h

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TOPIC 4: RELATIONSHIP BETWEEN STRESS AND STRAIN

Description:
4.1 Experimental study of the relationship between stress and strain.
4.2 Simple tensile test. Stress-strain curve. Hooke's law. Young's modulus. Poisson's ratio.
4.3 Generalised Hooke's law. Lamé's equations.
4.4 Applied exercises.

Specific objectives:
This topic highlights the need to use experimentation to gain an understanding of the laws governing the behaviour of materials, i.e. the laws of the relationship between stress and strain. We explain the simple tensile test and the data obtained from it. We also introduce Hooke's law, define Young's modulus and Poisson's ratio, and present the generalised Hooke's law and Lamé's equations, which together comprise all of the equations governing the linear elastic behaviour of deformable bodies. Finally, we introduce types of behaviour other than linear elastic behaviour.
The following exercises will be carried out as part of this topic: reflection on the meaning of Young's modulus, the shear modulus, Poisson's ratio, yield strength and fracture; and exercises involving the calculation of strain state using stress state and material properties.

Full-or-part-time: 15h
Theory classes: 3h
Practical classes: 3h
Self study : 9h

TOPIC 5: ELASTIC THEOREM

Description:
5.1 The general approach to the elastic problem
5.2 The elastic problem in shift. Navier's equations
5.3 The elastic problem in stress. Beltrami-Michell equations
5.4 Heat-induced stresses and strains. Duhamel's principle
5.5 Saint-Venant's principle
5.6 Applied exercises

Specific objectives:
The general approach to the elastic problem will be studied, through which it will be shown that there are as many equations as unknowns. Students will then go on to study the concept of shift, as expounded in Navier's equations and the Beltrami¿Michell equations. Heat-induced stresses and strains will also be dealt with in this topic. Finally, Saint-Venant's principle will be introduced, thanks to which the singularities at the points of application of the point-loads can be dispensed with for studying the effects in areas sufficiently far away from them.
In this topic, students will do exercises on the calculation of the state of stress and strain based on a given set of geometries, load states, the elastic characteristics of a material and boundaries in simple bodies. They will also do exercises to calculate the state of stress and strain based on a given set of geometries, load states, boundaries, and the elastic and thermal characteristics of geometrically simple bodies.

Full-or-part-time: 10h
Theory classes: 2h
Practical classes: 2h
Self study : 6h

TOPIC 6: WORK-ENERGY THEOREMS

Description:
6.1 Concept of internal energy or strain energy.
6.2 Relationship between external forces and the corresponding strain. Influence coefficients.
6.3 Expressions of internal energy.
6.4 Maxwell¿Betti reciprocal work theorem.
6.5 Castigliano's theorems.
6.6 Menabrea's theorem.
6.7 Principle of virtual work.
6.8 Applied exercises.

Specific objectives:
In this topic, we cover the energy theorems, exploring the various expressions of internal energy and studying the Maxwell¿Betti theorem, Castigliano's theorem and Menabrea's theorem, as well as the principle of virtual work. These theorems are especially relevant in strength of materials applications.
The following exercises will be carried out as part of this topic: reflection exercises involving the calculation of strain energy in bodies deformed under external loading; exercises involving the application of the Maxwell¿Betti reciprocal work theorem, Castigliano's theorem, Menabrea's theorem and the virtual work theorem in order to calculate displacement in cases with loads and geometries that are manageable at this level of the course.

Full-or-part-time: 30h
Theory classes: 6h
Practical classes: 6h
Self study : 18h

TOPIC 7: PLANE ELASTICITY IN CARTESIAN COORDINATES

Description:
7.1 State of plane strain. Strain tensors. Stress tensors. Principal stresses and directions
7.2 Equilibrium and boundary equations
7.3 Conditions of compatibility of strain tensors
7.4 State of plane stress. Stress tensors. Strain tensors. Principal stresses and directions. Mohr's circle. Lamé's ellipsoid
7.5 Equilibrium and boundary equations
7.6 Conditions of compatibility of strain tensors
7.7 Applied exercises

Specific objectives:
There are many applications of these concepts in engineering. This topic will concentrate on problems related to plane stress and strain. Students will therefore be expected to study such cases in depth. The topic will also cover plane stress and strain in Cartesian coordinates.

To consolidate their knowledge of this topic, students will do exercises based on real case studies, the significance of the state of plane strain and the state of plane stress. Practical exercises will be set on Mohr's circle for plane stress and strain. There will also be applied exercises on plane elasticity for calculating shifts, stress and strain tensors, applications of the principle of superposition, stress vectors in given orientations, strains and principal directions.

Full-or-part-time: 10h
Theory classes: 2h
Practical classes: 2h
Self study : 6h

TOPIC 8: FAILURE CRITERIA

Description:
8.1 Elastic failure criteria. General information.
8.2 Equivalent stress.
8.3 Maximum principal stress criterion.
8.4 Maximum longitudinal strain criterion.
8.5 Maximum shear stress criterion.
8.6 Maximum strain energy criterion.
8.7 Maximum distortion energy criterion.
8.8 Octahedral shear stress criterion.
8.9 Caquot's intrinsic curve theory.
8.10 Mohr¿Coulomb theory.
8.11 Concept of factor of safety. Allowable stresses.
8.12 Applied exercises.

Specific objectives:
This topic explores the various elastic failure criteria, providing students with the necessary background to know which criterion to consider for a given type of material. We introduce the concepts of equivalent stress for multiaxial states, allowable stress and factor of safety. This topic is included here because students, having covered the theory of elasticity, should become familiar with the theory's range of applicability in multiaxial cases.
As part of this topic, students will complete exercises that combine the calculation of material properties, geometry, factor of safety, and states of stress or of other external forces, all through simple designs of mechanical elements.

Full-or-part-time: 15h
Theory classes: 3h
Practical classes: 3h
Self study : 9h

TOPIC 9: EXPERIMENTAL METHODS IN ELASTICITY

Description:
9.1 Electrical extensometers. General information.
9.2 Extensometer gauges
9.3 Measuring instruments
9.4 Analysis of extensometer measurements
9.5 Load separation
9.6 The photo-elastic method. Fundamentals.
9.7 Neuman and Maxwell's laws
9.8 Plane polariscopes
9.9 Circular polariscopes
9.10 Separation of principal stresses
9.11 Applied exercises

Specific objectives:
In this last topic dealt with in the discipline of elasticity, experimental analysis methods of stress and strain will be examined. It will also cover electrical extensometers and the photo-elastic method, both of which have clear practical applications. This topic supplements the corresponding laboratory practicals.
In addition to the laboratory practicals, students will be set exercises designed to make them reflect on the most suitable type of extensometer (single point, midpoint, multipoint) or gauge (individual, rectangular rosette, delta rosette) to use for the determination of states of stress, principal directions and principal stresses at a point, as well as external forces in cases of single load.

Full-or-part-time: 15h
Theory classes: 3h
Practical classes: 3h
Self study : 9h

GRADING SYSTEM

1st exam: 40 %
2nd exam: 40 %
works: 20%
All those students who suspend, want to improve note or can not attend the partial exam, will have the opportunity to examine the same day of the final exam. If the circumstances do not make it feasible that it is the same day as the final exam, the teacher responsible for the subject will propose, via the Atenea platform, that the aforementioned recovery test will be carried out another day, during class hours .
The new note of the recovery exam will replace the old one only in case it is higher.

For those students who meet the requirements and submit to the reevaluation examination, the grade of the reevaluation exam will replace the grades of all the on-site written evaluation acts (tests, midterm and final exams) and the grades obtained during the course for lab practices, works, projects and presentations will be kept.
If the final grade after reevaluation is lower than 5.0, it will replace the initial one only if it is higher. If the final grade after reevaluation is greater or equal to 5.0, the final grade of the subject will be pass 5.0.

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Complementary:

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