320097 - AFED - Fourier Analysis and Differential Equations

Coordinating unit: 205 - ESEIAAT - Terrassa School of Industrial, Aerospace and Audiovisual Engineering
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2018
Degree: BACHELOR'S DEGREE IN AUDIOVISUAL SYSTEMS ENGINEERING (Syllabus 2009). (Teaching unit Compulsory)
ECTS credits: 6
Teaching languages: Catalan, Spanish

Teaching staff
Coordinator: Teresa Navarro Gonzalo
Others: Assumpta Sabater i Pruna

Prior skills
As a general rule it is considered highly desirable to have passed the first semester mathematics to be able to take the subject. Specifically, a basic knowledge of integral calculus is considered essential.

Degree competences to which the subject contributes

Specific:
1. (ENG) Capacitat per a la resolució dels problemes matemàtics que puguin platenjar-se a l'enginyeria. Aptitud per aplicar els coneixements sobre: àlgebra lineal; geometria, geometria diferencial; càlcul diferencial i integral; equacions diferenciales i amb derivades parciales; mètodes numèrics; algorítmica numèrica; estadística i optimització.

Transversal:
2. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.
3. SELF-DIRECTED LEARNING - Level 2: Completing set tasks based on the guidelines set by lecturers. Devoting the time needed to complete each task, including personal contributions and expanding on the recommended information sources.

Teaching methodology
- Face-to-face lecture sessions.
- Face-to-face practical work sessions.
- Independent learning and exercises.
- Preparation and completion of individual and/or group activities subject to assessment.

In the face-to-face lecture sessions, the lecturer will introduce the basic theory, concepts, methods and results for the subject and use examples to facilitate students' understanding.

Students will be expected to study in their own time so that they are familiar with concepts and are able to solve the exercises set, either manually or with the help of a computer.

The use of IT support tools will be encouraged: students will learn how to use a mathematical software package as a tool for performing numerical, symbolic and graphic calculations.

Learning objectives of the subject
320097 - AFED - Fourier Analysis and Differential Equations

Familiarise students with the techniques inherent to Fourier Analysis and with the interpretation of signals in the frequency range.

Familiarise students with some of the techniques used in Differential Equations, both Ordinary Equations and Partial Differential Equations. They will be expected to use the deterministic modelling tool and interpret its answers.

Teach students how to apply these techniques properly for solving common practical problems encountered by engineers. Use software tools for approaching and solving problems. Develop the specific and transversal competencies associated with the academic work.

**Study load**

<table>
<thead>
<tr>
<th>Total learning time: 150h</th>
<th>Hours large group: 30h</th>
<th>20.00%</th>
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<tbody>
<tr>
<td></td>
<td>Hours medium group:</td>
<td>20.00%</td>
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<tr>
<td></td>
<td>Hours small group:</td>
<td>0.00%</td>
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<td></td>
<td>Guided activities:</td>
<td>0.00%</td>
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<tr>
<td></td>
<td>Self study: 90h</td>
<td>60.00%</td>
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# Content

## Topic 1: FOURIER SERIES

<table>
<thead>
<tr>
<th>Description:</th>
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<tbody>
<tr>
<td>1.1. Numerical series.</td>
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<tr>
<td>1.2. Sequences and series of functions.</td>
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<tr>
<td>1.3. Fourier series.</td>
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<td>1.6. Complex expression. Parseval's identity</td>
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### Specific objectives:

For students to:
- Understand the concept of convergence of a numerical series, a sequence of functions and a series of functions.
- Understand the concept of Fourier series representation of a periodic signal and calculate it in real and exponential form.
- Understand the concepts of pointwise convergence vs. uniform convergence, Dirichlet's theorem and Gibbs phenomenon.
- Understand the concepts of spectrum and average signal strength.
- Understand and apply Parseval's theorem.
- Be able to calculate Fourier series and obtain the graphs of the partial sums and the spectra with the help of symbolic computation software.

### Learning time:

- Theory classes: 8h
- Practical classes: 8h
- Self study: 23h

## Topic 2: FOURIER TRANSFORM

<table>
<thead>
<tr>
<th>Description:</th>
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<tbody>
<tr>
<td>2.1. Deduction and spectrum. Parseval's identity</td>
</tr>
<tr>
<td>2.2. Properties of the Fourier transform.</td>
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<tr>
<td>2.3. Frequency description of LTI systems and filters.</td>
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### Specific objectives:

For students to:
- Understand the Fourier transform concept of a non-periodic signal and know how to calculate it.
- Understand the concepts of spectral energy density and Parseval's theorem.
- Understand the main properties of the Fourier transform: linearity, shifts, time scales and convolution.
- Understand the frequency description of LTI systems, in particular for describing filters.
- Calculate Fourier transforms and inverse Fourier transforms and use symbolic computation software to produce graphs of the corresponding spectra.

### Learning time:

- Theory classes: 4h 30m
- Practical classes: 4h 30m
- Self study: 16h
### Topic 3: ORDINARY DIFFERENTIAL EQUATIONS

<table>
<thead>
<tr>
<th>Learning time: 38h</th>
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<tbody>
<tr>
<td>Theory classes: 7h</td>
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<tr>
<td>Practical classes: 7h</td>
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<tr>
<td>Self study: 24h</td>
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**Description:**
3.1. General introduction to Ordinary Differential Equations (ODEs).
3.2. First-order linear ODEs.
3.3. Linear ODEs with constant coefficients.
3.4. Interpretation of results.

**Specific objectives:**
For students to:
- Understand the concept of ordinary differential equation and its solution. Understand the conditions for the existence and uniqueness of a solution to an initial value problem.
- Understand the concept of ODE problem modelling.
- Understand the techniques for the integration of simple ODEs, first-order linear ODEs (in particular the method of variation of parameters), and ODEs with constant coefficients.
- Use a linear ODE with constant coefficients to model the time description of an LTI system.
- Understand the concepts of resonance and stability.
- Solve ODEs and use symbolic computation software to obtain the corresponding graphs.

### Topic 4: LAPLACE TRANSFORM

<table>
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<th>Learning time: 22h</th>
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<tbody>
<tr>
<td>Theory classes: 4h</td>
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<tr>
<td>Practical classes: 4h</td>
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<tr>
<td>Self study: 14h</td>
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**Description:**
4.1. Definition of the Laplace Transform (LT)
4.2. Derivative theorem, initial value problems.
4.3. Properties of the LT.
4.4. The inverse LT.

**Specific objectives:**
For students to:
- Understand the concept of and calculate Laplace transforms.
- Understand the main properties of the Laplace transform: linearity, shifts, time scales and convolution, derivative theorems and initial and final value theorems.
- Solve Initial Value Problems (IVP) manually using the Laplace transform.
- Use symbolic computation software to calculate Laplace transforms and inverse Laplace transforms and to solve IVP with the Laplace transform.
## Topic 5: SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

**Description:**
5.1. Definition and examples. Solution and phase portraits.

**Related activities:**
For students to:
- Understand the concept of systems of differential equations and their importance in modelling specific phenomena.
- Understand the concept of the phase portrait.
- Understand phase portraits of planar linear systems and the role of the eigenvalues and eigenvectors of the coefficient matrix.
- Solve these systems using symbolic computation software.

**Learning time:** 13h
- Theory classes: 3h
- Practical classes: 3h
- Self study: 7h

## Topic 6: PARTIAL DIFFERENTIAL EQUATIONS

**Description:**
6.1. Definition and examples. Wave equation
6.2. Separation of variables and use of Fourier Series to solve Partial Differential Equations.

**Related activities:**
For students to:
- Understand the concept of partial differential equation.
- Understand the separation of variables method for solving the wave equation.
- Use symbolic computation software to obtain the corresponding graphs.

**Learning time:** 13h
- Theory classes: 3h 30m
- Practical classes: 3h 30m
- Self study: 6h
Planning of activities

<table>
<thead>
<tr>
<th>WRITTEN TESTS</th>
<th>Hours: 4h</th>
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<tbody>
<tr>
<td><strong>Description:</strong></td>
<td>Self study: 4h</td>
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<tr>
<td>Face-to-face individual tests in the timetable set by the School.</td>
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<tr>
<td><strong>Specific objectives:</strong></td>
<td></td>
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<tr>
<td>The student must have successfully achieved the objectives outlined in the contents that have been part of the corresponding test.</td>
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<table>
<thead>
<tr>
<th>OTHER ACTIVITIES</th>
<th>Hours: 10h</th>
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<tr>
<td><strong>Description:</strong></td>
<td>Self study: 10h</td>
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<tr>
<td>Tasks related to the subject contents.</td>
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<tr>
<td><strong>Specific objectives:</strong></td>
<td></td>
</tr>
<tr>
<td>The student must have successfully achieved the objectives outlined in the contents that have been part of the corresponding task.</td>
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Qualification system

It is evaluated by partial assessments with the following weights:
- 1st exam: 45%
- 2nd exam: 45%
- Tasks: 10%

The note of the 1st exam may be renewed with a second change examination, which will be done on the same date as the day set for the 2nd exam. Any matriculated student may be submitted. The final qualification of the 1st exam will be the highest mark between the 1st exam and the mark of the second change examination.

For those students who meet the requirements and submit to the re-evaluation examination, the grade of the re-evaluation exam will replace the grades of all the evaluation acts during the course.
If the final grade after re-evaluation is lower than 5, it will replace the initial one only if it is higher. If the final grade after re-evaluation is greater or equal to 5, the final grade of the subject will be pass 5.0.

To access the re-evaluation, students must have a final grade higher or equal to 2.0 but lower than 5.0 during the teaching period.

Regulations for carrying out activities

The assessment consists of the following acts of classroom assessment and/or other activities assessed as part of continuous assessment. If not done any of the events or activities will be considered qualified to zero.
### Bibliography

#### Basic:


#### Complementary:


#### Others resources:

- Lists of exercises for the course.
- Scripts for using MAPLE software to solve problems.