330502 - CAL1 - Calculus 1

Degree competences to which the subject contributes

Basic:
CB1. The students have demonstrated to possess and to understand knowledge in an area of study that starts from the base of the general secondary education, and is usually found to a level that, although it relies on advanced textbooks, also includes some aspects that involve knowledge from the vanguard of their field of study.
CB2. Students can apply their knowledge to their work or vocation in a professional way and possess the skills that are usually demonstrated through the elaboration and defense of arguments and problem solving within their area of study.

Specific:
CE1. Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge about: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and partial derivatives; numerical methods; numerical algorithms; statistics and optimization.

Generical:
CG3. Knowledge in basic and technological subjects that will enable them to learn new methods and theories and give them the versatility to adapt to new situations.

Transversal:
1. EFFICIENT ORAL AND WRITTEN COMMUNICATION - Level 1. Planning oral communication, answering questions properly and writing straightforward texts that are spelt correctly and are grammatically coherent.
2. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

Teaching methodology
MD1 Master class or lecture (EXP)
MD2 Problem solving and case study (RP)
MD5 Small-scale project, activity or assignment (PR)
MD6 Large-scale project or assignment (PA)
MD7 Assessment activities (EV)

Learning objectives of the subject
To learn basic mathematical principles and methods of single-variable calculus.
330502 - CAL1 - Calculus 1

The general aim is twofold: first students must acquire the knowledge required to understand, set up and solve basic mathematical engineering problems. The learning process should also help increase their ability to understand new methods and theories.

At the end of the Calculus 1 course, students should be able to do the following:
1. Set up and solve basic mathematical problems related to calculus of single-variable functions.
2. Apply the theoretical knowledge necessary for solving specific engineering problems and correctly interpreting the results.
3. Make elementary use of MATLAB for numeric and symbolic computations.

### Study load

<table>
<thead>
<tr>
<th>Study load</th>
<th>Hours large group:</th>
<th>Hours medium group:</th>
<th>Hours small group:</th>
<th>Guided activities:</th>
<th>Self study:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total learning time:</strong> 150h</td>
<td>30h</td>
<td>0h</td>
<td>30h</td>
<td>0h</td>
<td>90h</td>
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<tr>
<td></td>
<td>20.00%</td>
<td>0.00%</td>
<td>20.00%</td>
<td>0.00%</td>
<td>60.00%</td>
</tr>
</tbody>
</table>
## Content

<table>
<thead>
<tr>
<th>Topic 1: Single-variable differential calculus</th>
<th>Learning time: 50h</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theory classes: 10h</td>
</tr>
<tr>
<td></td>
<td>Laboratory classes: 10h</td>
</tr>
<tr>
<td></td>
<td>Self study: 30h</td>
</tr>
</tbody>
</table>

### Description:
Basic functions in engineering.
Limits and continuity. Theorems on continuous functions.
Derivation of single-variable functions.
Linear approximations.
Theorems on differentiable functions.
Relative and absolute extremes. Optimisation problems.
Zeros of functions: the bisection and Newton-Raphson methods.

### Related activities:
1, 2, 3, 4, 5

### Specific objectives:
At the end of this part students should be able, with the assistance of MATLAB in non-elementary cases, to do the following:

- Identify domains of functions and represent them graphically. Understand the concept of limits. Calculate limits of functions.
- Understand the concepts of continuity and differentiability.
- Identify whether a function is continuous or differentiable at one point.
- Calculate derivatives of functions defined either explicitly or implicitly.
- Write the equation of the tangent and the normal lines to a curve at a point.
- Write the linear approximation of a function at a point and use it to calculate approximately the function at nearby points.
- Understand the main theorems on continuous and differentiable functions, and apply them in concrete examples.
- Calculate relative and absolute extremes of a function.
- Set up and solve elementary optimisation problems.
- Understand and know how to use the bisection and Newton-Raphson methods for approximate calculation of roots of equations.
**Topic 2: Single-variable integral calculus**

**Learning time:** 50h  
Theory classes: 10h  
Laboratory classes: 10h  
Self study: 30h

**Description:**  
Riemann sums. The concept of definite integrals.  
The fundamental theorem of calculus.  
Calculation of primitives: immediate, by change of variable, by parts and rational.  
Calculation of areas of flat regions and volumes of revolution.  
Numerical integration: trapezoid and Simpson methods.  
Improper integrals.

**Related activities:**  
1, 2, 3, 4, 5

**Specific objectives:**  
At the end of this part students should be able, with the help of MATLAB in non-elementary cases, to do the following:

- Understand the concept of Riemann sums as a tool for defining magnitudes in engineering.  
- Calculate primitives and definite integrals.  
- Decompose a rational function into a sum of simple fractions.  
- Calculate areas of flat regions by integration.  
- Calculate volumes of solids generated by rotating a flat region around an axis that does not cut it.  
- Understand and know how to use the trapezium and Simpson methods for the approximate calculation of definite integrals.  
- Understand the concept of improper integrals and know how to calculate them in elementary cases.
### Topic 3: Series

<table>
<thead>
<tr>
<th>Learning time:</th>
<th>50h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory classes:</td>
<td>10h</td>
</tr>
<tr>
<td>Laboratory classes:</td>
<td>10h</td>
</tr>
<tr>
<td>Self study:</td>
<td>30h</td>
</tr>
</tbody>
</table>

#### Description:
- Numerical series. Convergence tests.
- Power series. Intervals of convergence.
- Representing functions with power series.
- Fourier series. Calculation of coefficients.
- Dirichlet's theorem.
- Complex expression of the Fourier series.

#### Related activities:
1, 2, 3, 4, 5

#### Specific objectives:
At the end of this part students should be able, with the help of MATLAB in non-elementary cases, to do the following:

1. Understand the concept of numerical series.
2. Identify the character of a numerical series.
3. Calculate the sum of particular series such as geometric or telescoping series.
4. Understand the concept of power series.
5. Find the interval of convergence of a power series.
6. Represent a function with a power series.
7. Understand the concept of Fourier series.
8. Calculate the trigonometric Fourier series of a periodic function.
10. Represent functions defined in a finite interval with a Fourier series.
11. Write the complex form of a Fourier series.
### Planning of activities

<table>
<thead>
<tr>
<th>Activity 1: Theory sessions</th>
<th><strong>Hours:</strong> 54h</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description:</strong></td>
<td></td>
</tr>
<tr>
<td>Presentation of the theory and applications of the various topics.</td>
<td></td>
</tr>
<tr>
<td><strong>Support materials:</strong></td>
<td></td>
</tr>
<tr>
<td>Documents in the Atenea virtual campus.</td>
<td></td>
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<tr>
<td>Compulsory reading list.</td>
<td></td>
</tr>
<tr>
<td><strong>Specific objectives:</strong></td>
<td></td>
</tr>
<tr>
<td>Those of topics 1, 2 and 3.</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Activity 2: Practice sessions</th>
<th><strong>Hours:</strong> 50h</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description:</strong></td>
<td></td>
</tr>
<tr>
<td>Setting up and solving problems.</td>
<td></td>
</tr>
<tr>
<td><strong>Support materials:</strong></td>
<td></td>
</tr>
<tr>
<td>Exercise statements.</td>
<td></td>
</tr>
<tr>
<td>Solved exercises (to be seen after students have attempted to solve them).</td>
<td></td>
</tr>
<tr>
<td>Documents in the Atenea virtual campus.</td>
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<tr>
<td>Online quizzes.</td>
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</tr>
<tr>
<td>Compulsory reading list.</td>
<td></td>
</tr>
<tr>
<td><strong>Specific objectives:</strong></td>
<td></td>
</tr>
<tr>
<td>Those of topics 1, 2 and 3.</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Activity 3: Lab sessions</th>
<th><strong>Hours:</strong> 26h</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description:</strong></td>
<td></td>
</tr>
<tr>
<td>Getting acquainted with the basic use of MATLAB.</td>
<td></td>
</tr>
<tr>
<td><strong>Support materials:</strong></td>
<td></td>
</tr>
<tr>
<td>Computer with MATLAB software.</td>
<td></td>
</tr>
<tr>
<td>Exercise statements.</td>
<td></td>
</tr>
<tr>
<td>Solved exercises (to be seen after students have attempted to solve them).</td>
<td></td>
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<tr>
<td>Documents in the Atenea virtual campus.</td>
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</tr>
<tr>
<td>Compulsory reading list.</td>
<td></td>
</tr>
<tr>
<td><strong>Specific objectives:</strong></td>
<td></td>
</tr>
<tr>
<td>Those of topics 1, 2 and 3.</td>
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</tbody>
</table>
## Activity 4: Written partial tests

**Description:**
Student learning tests, one for each of the three topics.

**Support materials:**
Test questions.

**Descriptions of the assignments due and their relation to the assessment:**
Individual answers.

**Specific objectives:**
Those of topics 1, 2 and 3.

<table>
<thead>
<tr>
<th>Hours</th>
<th>Theory classes: 3h</th>
<th>Self study: 9h</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total</strong></td>
<td>12h</td>
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</tbody>
</table>

## Activity 5: Lab partial tests

**Description:**
MATLAB knowledge assessment, one for each of the three topics.

**Support materials:**
Practice questions.

**Descriptions of the assignments due and their relation to the assessment:**
Individual answers.

**Specific objectives:**
Those of topics 1, 2 and 3.

<table>
<thead>
<tr>
<th>Hours</th>
<th>Laboratory classes: 2h</th>
<th>Self study: 6h</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total</strong></td>
<td>8h</td>
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</tbody>
</table>

## Activity 6 (additional): Final exam

**Description:**
A final exam including all topics.

**Support materials:**
Exam paper.

**Descriptions of the assignments due and their relation to the assessment:**
Individual answers.

**Specific objectives:**
Those of contents 1, 2 and 3.

<table>
<thead>
<tr>
<th>Hours</th>
<th>Theory classes: 3h</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total</strong></td>
<td>3h</td>
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</tbody>
</table>
Qualification system

The course mark, between 0 and 10, is obtained from the test results. The marks for the written partial tests (P1, P2 and P3) have a weighting of 25% each. The score of the lab partial tests (L) has a weighting of 25%. Thus, the course mark N is calculated as follows:

\[ N = 0.25 \times (P1 + P2 + P3) + 0.25 \times L \]

Students may additionally take a final exam to improve their mark in the written partial tests. The mark obtained from this test, G, gives rise to a new mark, NG:

\[ NG = 0.75G + 0.25L \]

The final mark is the highest between the course mark N and the mark NG.

Regulations for carrying out activities

Activities 1, 2, 3, 4 and 5 are compulsory for all students. Failure to do any of the tests in activities 4 or 5 will lead to a mark of zero for this test in calculating the values of N and NG.

Bibliography

Basic:


