200603 - PIPE - Probability and Stochastic Processes

Coordinating unit: 200 - FME - School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2019
Degree: MASTER'S DEGREE IN STATISTICS AND OPERATIONS RESEARCH (Syllabus 2013). (Teaching unit Optional)
ECTS credits: 5
Teaching languages: English

Teaching staff

Coordinator: JOSE FABREGA CANUDAS
Others: Segon quadrimestre:
        JOSE FABREGA CANUDAS - A

Prior skills

Students should be familiar with the topics covered in a first undergraduate course on probability. In particular, basic knowledge of the following subjects is assumed:

- Elementary probability theory.
- Basic probability models: binomial, geometric, Poisson, uniform, exponential, and normal distributions.

Concepts necessary to follow the course can be found, for example, in the following references:


Degree competences to which the subject contributes

Specific:

2. CE-2. Ability to master the proper terminology in a field that is necessary to apply statistical or operations research models and methods to solve real problems.
3. CE-3. Ability to formulate, analyze and validate models applicable to practical problems. Ability to select the method and / or statistical or operations research technique more appropriate to apply this model to the situation or problem.

Transversal:

1. TEAMWORK: Being able to work in an interdisciplinary team, whether as a member or as a leader, with the aim of contributing to projects pragmatically and responsibly and making commitments in view of the resources that are available.
The general aim of the course is to introduce the students to modelling of random phenomena. The course focuses on stochastic convergence problems that are crucial to statistics (laws of large numbers and central limit theorem) as well as on random processes (branching processes, random walks, Markov chains, the Poisson process). Tools related to transform methods (generating and characteristic functions) are also introduced. Special attention is given to the study of specific applications of the theoretical concepts.

Skills to be learned:
- Usage of probability and moment generating functions, and characteristic functions.
- To know the multivariate normal law and how to operate with jointly gaussian random variables.
- To understand the different modes of convergence of sequences of random variables as well as the precise meaning of the laws of large numbers and the central limit theorem.
- Basic concepts on stochastic processes.
- To work with Markov chains and the meaning of both stationary distributions and ergodic theorems.
- To understand the Poisson process.
- To identify probability models based on the theoretical results presented in the course.

**Study load**

<table>
<thead>
<tr>
<th>Total learning time: 125h</th>
<th>Hours large group: 30h</th>
<th>24.00%</th>
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<tbody>
<tr>
<td></td>
<td>Hours medium group: 0h</td>
<td>0.00%</td>
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<tr>
<td></td>
<td>Hours small group: 15h</td>
<td>12.00%</td>
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<td></td>
<td>Guided activities: 0h</td>
<td>0.00%</td>
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<tr>
<td></td>
<td>Self study: 80h</td>
<td>64.00%</td>
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Weekly class hours combine both theoretical and practical sessions. The theoretical lectures are devoted to a careful presentation of the fundamental concepts and the main results which are illustrated with examples. Some mathematical proofs are presented which, for their content and development, are particularly interesting from the learning and creative point of view. In the practical sessions the solution of a variety of exercises and problems is discussed.

Lists of exercises as well as guided work could be assigned to be carried out individually or in groups.

**Learning objectives of the subject**

The general aim of the course is to introduce the students to modelling of random phenomena. The course focuses on stochastic convergence problems that are crucial to statistics (laws of large numbers and central limit theorem) as well as on random processes (branching processes, random walks, Markov chains, the Poisson process). Tools related to transform methods (generating and characteristic functions) are also introduced. Special attention is given to the study of specific applications of the theoretical concepts.
## Content

<table>
<thead>
<tr>
<th>1. Generating Functions and Characteristic Function</th>
<th>Learning time: 14h 30m</th>
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</thead>
<tbody>
<tr>
<td><strong>Description:</strong></td>
<td>Theory classes: 3h</td>
</tr>
<tr>
<td>1.1 Probability and moment generating functions.</td>
<td>Laboratory classes: 1h 30m</td>
</tr>
<tr>
<td>1.2 The characteristic function.</td>
<td>Self study : 10h</td>
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<tr>
<td>1.3 Sum of a random number of independent random variables.</td>
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<td>1.4 Distributions with random parameters.</td>
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<tr>
<td>1.5 Application to the sample mean and sample variance.</td>
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<tr>
<th>2. Branching Processes</th>
<th>Learning time: 11h</th>
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<tbody>
<tr>
<td><strong>Description:</strong></td>
<td>Theory classes: 1h 30m</td>
</tr>
<tr>
<td>2.1 The Galton-Watson process.</td>
<td></td>
</tr>
<tr>
<td>2.2 Application to population growth.</td>
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<tr>
<td>2.3 Probability of ultimate extinction.</td>
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<tr>
<td>2.4 Probability generating function of the n-th generation.</td>
<td></td>
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</tbody>
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<table>
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<tr>
<th>3. The Multivariate Gaussian Distribution</th>
<th>Learning time: 16h</th>
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<tbody>
<tr>
<td><strong>Description:</strong></td>
<td>Theory classes: 4h 30m</td>
</tr>
<tr>
<td>3.1 Joint characteristic function of independent gaussian random variables.</td>
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<tr>
<td>3.2 The multidimensional gaussian law.</td>
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<tr>
<td>3.3 Linear transformations.</td>
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<tr>
<td>3.4 Linear dependence and singular gaussian distributions.</td>
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<tr>
<td>3.5 Multidimensional gaussian density.</td>
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</table>
### 4. Sequences of Random Variables

**Learning time:** 17h 30m  
Theory classes: 4h 30m  
Laboratory classes: 3h  
Self study: 10h  

**Description:**  
4.1 The weak law of large numbers. Convergence in probability.  
4.2 The central limit theorem. Convergence in distribution.  
4.3 Convergence in mean square.  
4.4 The strong law of large numbers. Almost-sure convergence.  
4.5 Borel Cantelli lemmas. Examples of application.  
4.6 Application to statistical estimation.

### 6. Random Walks

**Learning time:** 16h  
Theory classes: 4h 30m  
Laboratory classes: 1h 30m  
Self study: 10h  

**Description:**  
6.1 One-dimensional random walks.  
6.2 Returns to the origin.  
6.3 Random walks in the plane and the space.  
6.4 Introduction to brownian motion.

### 7. Markov Chains

**Learning time:** 25h  
Theory classes: 6h  
Laboratory classes: 3h  
Self study: 16h  

**Description:**  
7.1 Markov chains. The Markov property.  
7.2 Chapman-Kolmogorov equations.  
7.3 Recurrent and transient states.  
7.4 Absorbing chains.  
7.5 Stationary and limiting distributions.  
7.6 Application to Montecarlo methods.
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8. The Poisson Process

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<td>Theory classes: 6h</td>
</tr>
<tr>
<td>Laboratory classes: 3h</td>
</tr>
<tr>
<td>Self study: 16h</td>
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</tbody>
</table>

Description:
8.1 The Poisson process.
8.2 Intertransition times.
8.3 Birth and death processes.
8.4 Continuous time Markov chains.

Qualification system
The final grade (NF) will be calculated in the following manner:

\[
NF = \max(EF, 0.4*EF+0.4*EP+0.2*T)
\]

where EF is the final exam mark, EP is the partial exam mark and T is the mark of the exercises and assigned work throughout the course.

Bibliography

Basic:

Complementary:
- Sanz Solé, M. Probabilitats. Univ. de Barcelona, 1999.