220332 - Astrodynamics

Coordinating unit: 205 - ESEIAAT - Terrassa School of Industrial, Aerospace and Audiovisual Engineering
Teaching unit: 220 - ETSEIAT - Terrassa School of Industrial and Aeronautical Engineering
Academic year: 2019
Degree: MASTER'S DEGREE IN AERONAUTICAL ENGINEERING (Syllabus 2014). (Teaching unit Optional)
MASTER'S DEGREE IN SPACE AND AERONAUTICAL ENGINEERING (Syllabus 2016). (Teaching unit Optional)
ECTS credits: 5  Teaching languages: English

Teaching staff
Coordinator: De La Torre Sangrà, David
Others: De La Torre Sangrà, David

Opening hours
Timetable: To be defined

Prior skills
The student must have background knowledge of algebra, geometry, calculus, programming, basic physics (dynamics of point masses), basics orbital mechanics (two-body problem, Keplarian orbits, main perturbations produced by J2 according to first-order theory, orbital elements, patched conics, gravity assist, Hohmann transfer, basic impulsive maneuvers, launch geometry).

Requirements
The following subjects of the Degree are a prerequisite for this course: Calculus I, Linear Algebra, Physics I, Mechanics, Software Engineering, Space Engineering (or Introduction to Space Engineering of the first year of MUEA). The following subjects of MUEA are also a requirement: Aerodynamics, Flight and Orbital Mechanics and Software Engineering.

Degree competences to which the subject contributes

Specific:
CB06. Manage original concepts in research projects.
CB10. Improve self-learning capacity

CEEESPAC1. MUEA/MASE: Sufficient applied knowledge of the planning of space missions (specific competency for the specialisation in Space).
CEEESPAC2. MUEA/MASE: Advanced applied knowledge of orbital dynamics and space vehicle design (specific competency for the specialisation in Space).
CG06-MUEA. (ENG) Capacitat per a l'anàlisi i la resolució de problemes aeroespacials en entorns nous o desconeguts, dins de contextos amplis i complexos.
Learning objectives of the subject

The learning objectives of this subject consist in the development of the following skills:

- Ability to analyze the geometry of an orbit and its effects on the objectives of the mission.
- Ability to understand and develop a Delta-V (or amount of fuel) budget.
- Ability to solve Lambert’s problem (approach, selection of solution method, solution by iterative numerical algorithm, expression of the result in a form suitable for use in the description of the motion).
- Ability to study a planet-to-planet direct transfer and analyze the corresponding launch opportunities through the so-called Pork Chop Plot (PCP).
- Ability to determine the effects of a planetary gravity assist: given the initial heliocentric orbital elements, determine the effects of interaction with the given planet.
- Ability to calculate and interpret the Tisserand graph.
- Ability to distinguish among different optimization methods and their domains of applicability.
- Basic knowledge of the circular restricted problem of three bodies, their characteristics and peculiarities, their astrodynamical applications.
- Basic knowledge of design of low-thrust trajectory.

Study load

<table>
<thead>
<tr>
<th>Total learning time: 125h</th>
<th>Hours large group: 30h</th>
<th>24.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours small group:</td>
<td>15h</td>
<td>12.00%</td>
</tr>
<tr>
<td>Self study:</td>
<td>80h</td>
<td>64.00%</td>
</tr>
</tbody>
</table>
## Content

<table>
<thead>
<tr>
<th>MODULE 1 - GEOCENTRIC ORBITS</th>
<th>Learning time: 27h</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theory classes: 7h</td>
</tr>
<tr>
<td></td>
<td>Laboratory classes: 3h</td>
</tr>
<tr>
<td></td>
<td>Self study : 17h</td>
</tr>
</tbody>
</table>

**Description:**
This module consists in the following:
- Short reminder of fundamentals of orbital mechanics.
- Geocentric missions:
  - selection of orbital parameters versus mission objectives;
  - geometrical characteristics (groundtrack, coverage, azimuth-elevation, swath width, revisit time);
  - special orbits (Molniya, Tundra, geosynchronous, heliosynchronous, geostationary with the perturbations due to J22);
  - constellations (Walker code) and their characterization.

**Related activities:**
Activities 1 and 2

<table>
<thead>
<tr>
<th>MODULE 2 - LAMBERT'S PROBLEM</th>
<th>Learning time: 23h</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theory classes: 5h</td>
</tr>
<tr>
<td></td>
<td>Laboratory classes: 3h</td>
</tr>
<tr>
<td></td>
<td>Self study : 15h</td>
</tr>
</tbody>
</table>

**Description:**
The problem of Lambert, also known as the two-point boundary value problem, is a classical one in astrodynamics and celestial mechanics. Its importance is due to its central role in a large number of issues. This is why it is assigned an entire module in this course. The topic will be addressed following a traditional approach:
- Problem definition, statement of the theorem of Lambert, sketch of proof
- Numerical aspects of the problem
- Classical methods of solution (Lagrange, Gauss) with exercises
- Introduction to modern methods
- Application to the problem of interplanetary transfer. Examples
- Employment for studying the opportunities for such transfers. Examples.
- Pork Chop Plot (PCP)
### MODULE 3 - INTERPLANETARY TRAJECTORIES

**Description:**
- Discussion on dynamical models: two-body problem, N-body problem, patched conics
- Rigorous definition of gravitational sphere of influence (SOI)
- Calculation of the intersection of a planet-to-planet trajectory with the planetary SOI
- Physics of the gravity assist (swingby) in 2D and 3D
- Effects of gravity assist on the heliocentric orbital elements
- Characterization of the planets as good or bad accelerators. Examples.
- Introduction to the optimization of interplanetary trajectories

**Learning time:** 33h
- Theory classes: 8h
- Laboratory classes: 4h
- Self study: 21h

### MODULE 4 - THE THREE-BODY PROBLEM

**Description:**
The three-body problem in the discussion on the dynamical systems and their approximations. General problem of three bodies.
- Approximation: the circular restricted problem of three bodies.
- Dynamical equations in the synodical reference frame, equilibria, Jacobi constant, zero velocity surfaces
- Linear stability of the equilibrium points
- Periodic and non-periodic orbits around the collinear points
- Invariant manifolds
- Application to the study of trajectories of small bodies and space missions.

**Learning time:** 23h
- Theory classes: 6h
- Laboratory classes: 3h
- Self study: 14h

### MODULE 5 - INTRODUCTION TO THE DESIGN OF LOW-THRUST TRAJECTORIES

**Description:**
Low-thrust trajectories (electric propulsion or solar sail) as control problems.
Introduction to optimal control theory:
- Principle of Pontryagin
- Types of boundary conditions
- Analysis and solution of simple cases
Importance in Astrodynamics. Applications

**Learning time:** 19h
- Theory classes: 4h
- Laboratory classes: 2h
- Self study: 13h
The final grade is computed by means of the following formula:

\[ G_{\text{final}} = 0.1 \times G_{\text{DE1}} + 0.1 \times G_{\text{DE2}} + 0.4 \times G_{\text{PRJ}} + 0.4 \times G_{\text{FEX}} \]

where \( G \) means grade and
- \( DE1 \) = deliverable exercise
- \( DE2 \) = deliverable exercise
- \( PRJ \) = course project
- \( FEX \) = final exam

More information in the Activities section.

For those students who meet the requirements and submit to the reevaluation examination, the grade of the reevaluation exam will replace the grades of all the on-site written evaluation acts (tests, midterm and final exams) and the grades obtained during the course for lab practices, works, projects and presentations will be kept.

If the final grade after reevaluation is lower than 5.0, it will replace the initial one only if it is higher. If the final grade after reevaluation is greater or equal to 5.0, the final grade of the subject will be pass 5.0.

**Regulations for carrying out activities**

In the in-class exams (i.e., FEX) lecture notes, slides and formula sheets are not permitted. The use of non-programable calculator is allowed. The duration of each in-class exam is of one hour. The exam done from home (i.e., DE1 and DE2) must be carried out individually: the students will receive the text of the exam through Atenea and will be given a deadline (typically of a few days, the exact time depending on the characteristics of the assignment) for the delivery of a computer code that solves the proposed problem and a report that illustrates it.

**Bibliography**

**Basic:**