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230857 - NMC - Numerical Methods for Continuum Systems

Coordinating unit: 230 - ETSETB - Barcelona School of Telecommunications Engineering
Teaching unit: 748 - FIS - Department of Physics
Academic year: 2019
Degree: MASTER'S DEGREE IN ENGINEERING PHYSICS (Syllabus 2018). (Teaching unit Optional)
ECTS credits: 4 Teaching languages: English

Teaching staff

Coordinator: Juan Sánchez Umbría

Others: Fernando Mellibovsky Elstein

Prior skills

Knowledge at the level of a degree of Linear Algebra, Calculus, Differential Equations, Numerical Methods and General Physics.

Degree competences to which the subject contributes

Basic:
CB6. (ENG) Poseer y comprender conocimientos que aporten una base u oportunidad de ser originales en el desarrollo y/o aplicación de ideas, a menudo en un contexto de investigación
CB7. (ENG) Que los estudiantes sepan aplicar los conocimientos adquiridos y su capacidad de resolución de problemas en entornos nuevos o poco conocidos dentro de contextos más amplios (o multidisciplinares) relacionados con su área de estudio.
CB8. (ENG) Que los estudiantes sean capaces de integrar conocimientos y enfrentarse a la complejidad de formular juicios a partir de una información que, siendo incompleta o limitada, incluya reflexiones sobre las responsabilidades sociales y éticas vinculadas a la aplicación de sus conocimientos y juicio.

Teaching methodology

- Lectures.
- Problem solving or programming exercises sessions.
- Individual assignments to solve exercises or to code numerical methods.
- Programming sessions of numerical methods.

Learning objectives of the subject

To learn the fundamentals, implementation and the applications of the numerical methods for the partial differential equations of the Mathematical Physics; in particular of the Finite Element methods.

After the course the student must be able to:
- Write the weak form of the differential equations.
- Mesh the computational domain.
- Choose an adequate type of Finite Element for a particular problem.
- Complete the discretization of the problem.
- Write an efficient code to solve the problem.
- Interpret the results and estimate the error of the solution.
- Use a standard Finite Elements library.

Study load

Total learning time: 100h	Hours large group:	36h	36.00%
	Self study:	64h	64.00%

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Content

Introduction.	Learning time: 5h Theory classes: 2h Self study : 3h

Description:
In this brief introduction the different methods to discretize the equations of Mathematical Physics will be exposed: finite differences, finite elements, and finite volumes.

Weak formulation of differential equations, Galerkin and collocation methods.	Learning time: 6h Theory classes: 2h Self study : 4h

Description:
It will be explained how to obtain the weak formulation of differential equations, used in the finite elements method, and the different ways to obtain the approximated equations. The weak formulation of several Physics equations (Thermodynamics, Elasticity, Fluid Mechanics, Electromagnetism, etc.) will be written. The Galerkin, Petrov-Galerkin and collocation projections of several equations of Physics will be described.

The Finite Element method.	Learning time: 11h Theory classes: 4h Self study : 7h

Description:
The objective of this chapter is to introduce the different types of finite elements. It will cover the nodal and modal formulations, the piece-wise Lagrangian approximation in triangles and quadrilaterals, the isoparametric mapping, other types of elements with higher continuity across elements, or other requirements, needed for particular problems. The interpolation errors, and the concepts of h, p, and hp convergence will also be studied.

Implementation of the Finite Element Method.	Learning time: 16h Theory classes: 6h Self study : 10h

Description:
The practical implementation of the FEM will be studied in this chapter, in order to write efficient numerical codes . This includes knowing how to mesh a domain, using for instance open source grid generators as Gmsh, the assembly of the matrices and vectors associated with the linear operators and forcing terms in the equations, using or not quadrature formulas, and estimating the error of the final solutions in some examples. Examples of application developing the full code from scratch in a programming language as Octave to facilitate the graphical representation or using a high-level FEM library as FEniCS will be studied in detail.

Complements of Numerical Linear Algebra and of non-linear systems of equations.	Learning time: 9h Theory classes: 3h Self study : 6h

Description:
Depending on the previous knowledge of the students it will be necessary to spend some time describing some numerical techniques of Linear Algebra. In particular on
matrix storage for sparse matrices, and computational methods for high-dimensional linear systems and eigenvalue problems. The solution of nonlinear systems of equations, and the study of the dependence of the solutions with the parameters of the problem will also be treated.

Finite Elements libraries. Introduction to FEniCS-Python.	Learning time: 23h Theory classes: 8h Self study : 15h

Description:
The use of the FEM library FEniCS-Python will be explained in detail with special emphasis in the application to several Physics problems. The students will have to present individual or small group assignments using this library.

Time integration.	Learning time: 11h Theory classes: 4h Self study : 7h

Description:
The solution of time evolution problems (advection-diffusion, wave equations, Navier-Stokes, etc.) will be studied with special atention to the stability of the numerical time-stepping schemes. Schemes of total discretization, method of lines, operator splitting, etc. will be considered.

Introduction to finite volumes and discontinuous Galerkin methods.	Learning time: 8h Theory classes: 3h Self study : 5h

Description:

The limitations of the FEM for the solution of advection-diffusion problems at high Peclet number, for the treatment of hyperbolic equations, etc. will be exposed together with their possible solution by means of the methods giving name to this chapter.

High order methods.	Learning time: 11h Theory classes: 4h Self study : 7h

Description:

This chapter is an introduction to the spectral elements method which allows reaching a high level of accuracy in space, and the time-stepping algorithms which allow the same in time. The spectral elements can be introduced in the chapter of the FEM without the need of a specific chapter, if details can be avoided. The same holds for the high-order time steppers, which can be seen as particular types of lines methods.

Qualification system

- Evaluation of the programming assignments (including oral presentation) (PA): 70% of the final mark.
- Evaluation of home exercises (oral presentation) (PS): 30% of the final mark.

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Bibliography

Basic:

Larson, M.G.; Bengzon, F. The finite element method: theory, implementation and applications. Berlin: Springer, 2013. ISBN 9783642332869.

Langtangen, H.P.; Logg, A. Solving PDEs in Python: the FEniCS tutorial I. Cham: Springer, 2016. ISBN 9783319524610.

Johnson, C. Numerical solution of partial differential equations by the finite element method. Mineola, NY: Dover Publications, 2009. ISBN 9780486469003.

Gockenbach, M.S. Understanding and implementing the finite element method. Philadelphia: SIAM, Society for Industrial and Applied Mathematics, 2006. ISBN 9780898716146.

Complementary:

Braess, D. Finite elements: theory, fast solvers, and applications in solid mechanics. Cambridge: Cambridge University Press, 2007. ISBN 9780521705189.

Zienkiewicz, O.C.; Morgan, K. Finite elements and approximation. Mineola, NY: Dover publications, 1983. ISBN 978-0486453019.

Zienkiewicz, O.C.; Taylor, R.L.; Zhu, J.Z. The finite element method: its basis & fundamentals. 7th ed. Amsterdam: Elsevier Butterworth-Heinemann, 2013. ISBN 9781856176330.

Elman, H.C.; Silvester, D.J.; Wathen, A.J. Finite elements and fast iterative solvers: with applications in incompressible fluid dynamics. 2nd ed. Oxford: Oxford University Press, 2014. ISBN 9780199678808.