Degree competences to which the subject contributes

Specific:
1. The student will be able to analyze and design linear systems (single and multiple variables, external and internal representation) and nonlinear systems. This includes their stability, controller design and evaluation of closed-loop response.
2. The student will be able to use analysis tools and computer-aided design of control systems in the tasks usual analysis, simulation and controller design.

General:
3. Have adequate mathematical skills, analytical, scientific, instrumental, technological, and management information.
4. Ability to conduct research, development and innovation in the field of systems engineering, control and robotics, and as to direct the development of engineering solutions in new or unfamiliar environments, linking creativity, innovation and transfer of technology.

Teaching methodology

The methodology of the course combines face to face instruction such as master classes and practical sessions with self-study through the development of problem assignments. These methodologies are detailed in the activities description section.

Learning objectives of the subject

By the end of the course, the students will be able to:
- Acquire and use the basis knowledge and fundamentals for the design of predictive controllers based on mathematical models of the given systems.
- Design different structures of linear predictive controllers taking into account the main (dynamic and/or static) features of the system under study.
- Suitably modify the predictive controller designs in such a way that those designs fit the current requirements and technological challenges.
240AR054 - Model-Based Predictive Control and Hybrid Systems

- Identify different designs of advanced predictive controllers (e.g., hybrid MPC, nonlinear MPC, explicit MPC), and recognise their main features as well as their fundamentals.
- Enumerate the main hybrid system frameworks and describe their features.
- Apply hybrid system modelling techniques.
- Analyse hybrid systems to determine their properties and qualities.
- Use software tools for the modelling, simulation, analysis and control of hybrid systems.

### Study load

<table>
<thead>
<tr>
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<th>Total learning time: 112h 30m</th>
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<tbody>
<tr>
<td></td>
<td>Hours medium group: 27h</td>
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<tr>
<td></td>
<td>Hours small group: 13h 30m</td>
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<td></td>
<td>Self study: 72h</td>
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</tbody>
</table>

- Total learning time: 112h 30m
- Hours medium group: 27h (24.00%)
- Hours small group: 13h 30m (12.00%)
- Self study: 72h (64.00%)
# 1. Introduction to hybrid systems

| Description: | 1.1 Hybrid system features.  
1.2 Examples of hybrid systems.  
1.3 Modelling formalisms. |
| Related activities: | Master classes, problem assignments, final examination |
| Specific objectives: | The student should be able to define a hybrid system and become aware of the complexity of the techniques needed to analyse them. Moreover, he should be able to recognise different existing formalisms that mathematically describe them. |

## Learning time: 4h 15m
- Theory classes: 2h 15m
- Self study: 2h

## Related activities:
- Master classes, problem assignments, final examination

## Specific objectives:
- The student should be able to define a hybrid system and become aware of the complexity of the techniques needed to analyse them. Moreover, he should be able to recognise different existing formalisms that mathematically describe them.

# 2. Hybrid automata

| Description: | 2.1 Mathematical representation  
2.2 Classes of hybrid automata  
2.3 Properties  
2.4 Verification techniques |
| Related activities: | Master classes, practical sessions, problem assignments, final examination |
| Specific objectives: | The student should be able to model a hybrid system as a hybrid automaton and recognise its specific class. Moreover, he should be able to determine its particular properties based on its executions. He should also be able to apply different verification techniques to hybrid automata. Lastly, he should become familiar with existing software packages that support the modelling, simulation and analysis of hybrid automata. |

## Learning time: 26h
- Theory classes: 6h 30m
- Practical classes: 2h 30m
- Self study: 17h

## Related activities:
- Master classes, practical sessions, problem assignments, final examination

## Specific objectives:
- The student should be able to model a hybrid system as a hybrid automaton and recognise its specific class. Moreover, he should be able to determine its particular properties based on its executions. He should also be able to apply different verification techniques to hybrid automata. Lastly, he should become familiar with existing software packages that support the modelling, simulation and analysis of hybrid automata.
3. Mixed logical dynamical systems

**Description:**
- 3.1 Mathematical representation of MLD systems
- 3.2 Other equivalent formalisms: DHA, PWA.
- 3.3 Verification techniques

**Related activities:**
Master classes, practical sessions, problem assignments, final examination

**Specific objectives:**
The student should be able to model a hybrid system as a MLD, DHA or PWA system. He should also be able to apply optimization techniques to the verification of this class of hybrid systems. Lastly, he should become familiar with existing software packages that support the modelling, simulation and analysis of this class of hybrid systems.

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**Learning time:** 6h 15m
- Theory classes: 2h 15m
- Self study: 4h

**Description:**
- 4.1 Classic control vs. Modern control
- 4.2 Industrial features and statistics
- 4.3 The basic idea and MPC fundamentals

**Related activities:**
Master classes, final examination

**Specific objectives:**
The student will be able to identify the main features of the MPC and its role nowadays within the industrial and scientific/academic framework. Moreover, the student will also be able to describe both the main fundamental parts of the MPC design and the basic idea of the whole strategy.
## 5. Linear Model Predictive Control

**Description:**
- 5.1 Basic formulation
  - 5.1.1 Elements
  - 5.1.2 Finite and infinite prediction/control horizons
  - 5.1.3 Constraints handling (hard, soft and time-varying constraints)
- 5.2 Solution of the predictive control problem
- 5.3 Feasibility and Stability
  - 5.3.1 Terminal constraints
  - 5.3.2 Model uncertainty
  - 5.3.3 Robustness analysis
  - 5.3.4 Iterative feasibility

**Related activities:**
Master classes, practical sessions, problem assignments, final examination

**Specific objectives:**
The student will be able to formulate a predictive control problem, defining each part properly, as well as describing the tuning parameters of the controller and the way of determining them. Similarly, the student will acquire the ability of solving the MPC problem posed. Moreover, other abilities related to the description and enhancement of the performance related to a closed loop using MPC will also be discussed and acquired.

### Learning time:
- Theory classes: 6h 30m
- Practical classes: 2h 30m
- Self study: 14h 30m

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## 6. Limitations of the MPC strategy

**Description:**
- 6.1 Complexity
- 6.2 Feasibility and existence of solutions of the optimisation problem
- 6.3 Computational requirements and software tools
- 6.4 Real-time implementations (fast MPC approaches)
- 6.5 Large-scale systems

**Related activities:**
Master classes, problem assignments, final examination

**Specific objectives:**
The student will be able to determine the limits of the predictive control designs, finding possible solutions and alternative strategies to overcome them. Moreover, the student will also be able to identify the relationship between the nature of the real system and the family of MPC controllers suitable for the resultant closed-loop scheme.

### Learning time:
- Theory classes: 2h 15m
- Self study: 7h
7. Other MPC approaches

<table>
<thead>
<tr>
<th>Description:</th>
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<tbody>
<tr>
<td>7.1 Explicit MPC</td>
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<tr>
<td>7.1.1 Invariant and feasible sets</td>
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<tr>
<td>7.1.2 Multi-parametric programming</td>
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<tr>
<td>7.2 Nonlinear MPC</td>
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<tr>
<td>7.2.1 General formulation</td>
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<tr>
<td>7.2.2 Solving algorithms</td>
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<tr>
<td>7.2.3 Basic examples</td>
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<tr>
<td>7.3 Hybrid MPC</td>
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<tr>
<td>7.3.1 General formulation</td>
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<tr>
<td>7.3.2 Solving algorithms</td>
</tr>
<tr>
<td>7.3.3 Basic examples</td>
</tr>
</tbody>
</table>

**Related activities:**
Master classes, practical sessions, problem assignments, final examination

**Specific objectives:**
The student will be able to determine the explicit solution of an MPC problem through the use of multi-parametric optimisation and the computation of invariant and feasible sets. Moreover, the student will also be able to recognise concepts and basic tools for the statement of MPC problems taking into account different types of system models such as nonlinear and hybrid ones.
### Planning of activities

#### 1. MASTER CLASSES

**Hours:** 28h  
Theory classes: 28h

**Description:**  
The professor will explain the theory and introduce short exercises to improve the understanding of the subject, which will be solved by the students in class. Active participation of the students will be encouraged by the professor. Sessions of 2h/3h per week will be scheduled for this activity.

**Support materials:**  
Handouts, bibliography and short exercise assignments.

**Descriptions of the assignments due and their relation to the assessment:**  
Exercises solved in class

#### 2. PRACTICAL SESSIONS

**Hours:** 26h  
Practical classes: 10h  
Self study: 16h

**Description:**  
Numerical computer tools (such as MATLAB) will be used for the simulation and analysis of hybrid systems and the design of MPC controllers. Practical assignments will be solved in pairs. Sessions of 2.5h per week will be scheduled for this activity.

**Support materials:**  
Practical assignments, software tool user's guides, handouts and bibliography.

**Descriptions of the assignments due and their relation to the assessment:**  
Practical assignment reports

#### 3. PROBLEM ASSIGNMENTS

**Hours:** 20h  
Self study: 20h

**Description:**  
Autonomous problem solving that involve theoretical and practical development, and the use of numerical computer tools. Problem assignments will be solved individually.

**Support materials:**  
Problem assignments, software tool user's guides, handouts and bibliography.

**Descriptions of the assignments due and their relation to the assessment:**  
Problem assignment reports

#### 4. FINAL EXAMINATION

**Hours:** 38h 30m  
Theory classes: 2h 30m  
Self study: 36h

**Description:**  
An exam will be scheduled at the end of the course, which will assess the full course contents. It will be an open-book written exam that will be solved individually, possibly with the aid of numerical computer tools.
Support materials:
Exam, handouts and bibliography.

Descriptions of the assignments due and their relation to the assessment:
Filled out exam form.

Qualification system

The acquired competences and capabilities will be assessed on the basis of three activities:
- practical assignment reports (30% of the course score)
- problem assignment reports (30% of the course score)
- final examination (40% of the course score)
Bibliography

Basic:


Complementary:


Others resources:

Hyperlink

Bemporad, A., Modeling, Control and Reachability Analysis of Discrete-time Hybrid Systems, Lecture Notes \textsuperscript{1} DISC School on Hybrid Systems, Mar. 2003.

http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.76.2926&rep=rep1&type=pdf