Degree competences to which the subject contributes

Specific:
8378. Practical numerical modeling skills. Ability to acquire knowledge on advanced numerical modeling applied to different areas of engineering such as: civil or environmental engineering or mechanical and aerospace engineering or bioengineering or Nanoengineering and naval and marine engineering, etc.
8380. Materials modeling skills. Ability to acquire knowledge on modern physical models of the science of materials (advanced constitutive models) in solid and fluid mechanics.
8382. Experience in numerical simulations. Acquisition of fluency in modern numerical simulation tools and their application to multidisciplinary problems engineering and applied sciences.
8383. Interpretation of numerical models. Understanding the applicability and limitations of the various computational techniques.
8384. Experience in programming calculation methods. Ability to acquire training in the development and use of existing computational programs as well as pre and post-processors, knowledge of programming languages ??and of standard calculation libraries.
This course is the student's first approach to the basic concepts of the Finite Element Method, including the basic formulation, analysis of the obtained methods, and essential aspects of their implementation. The presentation is carried out using linear problems with practical applications as heat transfer and elasticity, complemented by practical exercises.

- The student will be able to understand and assimilate the foundations of the linear analysis with finite elements, obtain the weak form of the variational formulation and its solution, as well as know the basic structure of a finite elements program.
- Comprehend why the method of the finite elements approximates to the solution of the PDE, considering the errors of truncation, consistency, convergence, and stability of the solution of a linear system of equations and the eigenvalue problems.
- Manually solve linear problems of mechanics and transfer of heat using FE and simultaneously be able to use properly an FE code to obtain solutions for engineering problems.
- The students will also develop the capacity of abstraction and synthesis, understand the structure of vectorial spaces and the concept of linearity.
- The students will have to acquire independence in their studies, be able to implement and use computer programs and rationally interpret the results obtained.

Introduction to the finite elements methods
- Errors
- Linear Systems of equations
- Approximation and interpolation
- Numerical Integration
- Discrete Systems and continuous systems. Introduction to the FEM
- Solving with FEM of one-dimensional problems.
- Application to Poisson equation
- More advanced one-dimensional finite Elements
- Application of the FEM to the equation of Poisson in two dimensions
- Application of the FEM to the equation of Poisson in three dimensions
- Matrix Formulation for the solution of Poisson's problem through the FEM
- Obtaining of shape functions for two and three-dimensional class C0 elements
- Two and three-dimensional isoparametric elements
- Two-dimensional elasticity problems

Learning resources:
- Zienkiewicz, O.C.; Morgan, K., Finite elements and approximation, Wiley, 1983
- Huerta, A.; Sarrate, J.; Rodríguez-Ferran, A. Métodos numéricos.
Introducción, aplicaciones y programación. Edicions UPC, 1999
- Trefethen, L.N. & Bau, D., Numerical linear algebra, SIAM, 1996

### Study load

<table>
<thead>
<tr>
<th>Total learning time: 125h</th>
<th>Theory classes:</th>
<th>15h</th>
<th>12.00%</th>
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<tbody>
<tr>
<td></td>
<td>Practical classes:</td>
<td>15h</td>
<td>12.00%</td>
</tr>
<tr>
<td></td>
<td>Laboratory classes:</td>
<td>7h 30m</td>
<td>6.00%</td>
</tr>
<tr>
<td></td>
<td>Guided activities:</td>
<td>7h 30m</td>
<td>6.00%</td>
</tr>
<tr>
<td></td>
<td>Self study:</td>
<td>80h</td>
<td>64.00%</td>
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</tbody>
</table>
## Introduction

<table>
<thead>
<tr>
<th>Learning time: 4h 48m</th>
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<tbody>
<tr>
<td>Theory classes: 1h</td>
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<tr>
<td>Practical classes: 1h</td>
</tr>
<tr>
<td>Self study : 2h 48m</td>
</tr>
</tbody>
</table>

**Description:**
- Problems to be solved: Boundary problems and Initial value problems.
- PDEs, boundary conditions and initial conditions
- Elliptic, parabolic and hyperbolic problems
- Balance and equilibrium.

Examples of physical problems.

## Basic concepts of FEM

<table>
<thead>
<tr>
<th>Learning time: 9h 36m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory classes: 1h</td>
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<tr>
<td>Practical classes: 2h</td>
</tr>
<tr>
<td>Laboratory classes: 1h</td>
</tr>
<tr>
<td>Self study : 5h 36m</td>
</tr>
</tbody>
</table>

**Description:**
- Type of approximation: piecewise polynomials / spline
- Strong form
- Weighted residuals: dealing with boundary conditions
- Point collocation, Least squares and Galerkin formulations
- Weak form

Examples
Problems

## Poisson problem in 1D

<table>
<thead>
<tr>
<th>Learning time: 9h 36m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory classes: 2h</td>
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<tr>
<td>Practical classes: 1h</td>
</tr>
<tr>
<td>Laboratory classes: 1h</td>
</tr>
<tr>
<td>Self study : 5h 36m</td>
</tr>
</tbody>
</table>

**Description:**
- Space and functional discretizations
- Weak form: resulting linear system

Examples
Problems
### Linear Elasticity 1D

**Description:**
- Elementary stiffness matrix
- Assembly in 1D
- Prescribed displacements

Isoparametric elements

Examples

**Learning time:** 9h 36m
- Theory classes: 2h
- Practical classes: 1h
- Laboratory classes: 1h
- Self study: 5h 36m

---

### Poisson problem in 2D and 3D

**Description:**
- Basic equations
- Weak form
- FEM discretization
- Boundary conditions

Examples

**Learning time:** 7h 11m
- Theory classes: 2h
- Practical classes: 1h
- Self study: 4h 11m

---

### Higher order elements

**Description:**
- 2D/3D elements
- Isoparametric elements
- Numerical integration
- General solution of Poisson equation with FEM

Examples

**Learning time:** 9h 36m
- Theory classes: 2h
- Practical classes: 2h
- Self study: 5h 36m
### Linear Elasticity 2D and 3D

**Learning time:** 12h

- Theory classes: 2h
- Practical classes: 2h
- Laboratory classes: 1h
- Self study: 7h

**Description:**
- Basic equations
- Principle of virtual work (PVW)
- FEM discretization

**Examples**

### Error estimates and mesh adaptivity

**Learning time:** 9h 36m

- Theory classes: 2h
- Practical classes: 2h
- Self study: 5h 36m

**Description:**
- A priori error estimates: convergence ratio
- A posteriori error estimates
- Mesh adaptivity

**Examples**

### Transient problems

**Learning time:** 9h 36m

- Theory classes: 1h
- Practical classes: 2h
- Laboratory classes: 1h
- Self study: 5h 36m

**Description:**
- Space-time splitting: the method of lines
- Direct time integration, stability
- Newmark method
- Modal analysis

**Examples**

**Problems**
The mark of the course is obtained from the ratings of continuous assessment and the evaluation test.

Continuous assessment consist in several homeworks, carried out during the year.

The evaluation test consists of a part with questions about concepts associated with the learning objectives of the course with regard to knowledge or understanding, and a part with a set of application exercises.

Failure to perform the continuous assessment activity in the scheduled period will result in a mark of zero in that activity.

Basic: