Date: 29/11/2022

Course guide
250971 - 250971 - Reduced Order Modelling

Last modified: 21/11/2022

Unit in charge: Barcelona School of Civil Engineering
Teaching unit: 751 - DECA - Department of Civil and Environmental Engineering.

Degree: MASTER'S DEGREE IN NUMERICAL METHODS IN ENGINEERING (Syllabus 2012). (Optional subject).

Academic year: 2022 ECTS Credits: 5.0 Languages: English

LECTURER

Coordinating lecturer: PAVEL RYZHAKOV

Others: JOSÉ RAÚL BRAVO MARTÍNEZ, ALBERTO GARCIA GONZALEZ, MOHAMMAD REZA HASHEMI, PAVEL RYZHAKOV, SERGIO ZLOTNIK MARTINEZ

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
8382. Experience in numerical simulations. Acquisition of fluency in modern numerical simulation tools and their application to multidisciplinary problems engineering and applied sciences.
8383. Interpretation of numerical models. Understanding the applicability and limitations of the various computational techniques.
8384. Experience in programming calculation methods. Ability to acquire training in the development and use of existing computational programs as well as pre and post-processors, knowledge of programming languages and of standard calculation libraries.

TEACHING METHODOLOGY

The course consists of 1.2 hours per week of classroom activity (large size group) and 1.2 hours weekly with half the students (medium size group).

The 1.2 hours in the large size groups are devoted to theoretical lectures, in which the teacher presents the basic concepts and topics of the subject, shows examples and solves exercises.

The 1.2 hours in the medium size groups is devoted to solving practical problems with greater interaction with the students. The objective of these practical exercises is to consolidate the general and specific learning objectives.

The rest of weekly hours devoted to laboratory practice.

Support material in the form of a detailed teaching plan is provided using the virtual campus ATENEA: content, program of learning and assessment activities conducted and literature.

Although most of the sessions will be given in the language indicated, sessions supported by other occasional guest experts may be held in other languages.
LEARNING OBJECTIVES OF THE SUBJECT

The students will acquire the knowledge of the techniques of reduction of models of types a posteriori and a priori. They will apply the different techniques of ROM (Reduced order modeling) to general data problems, as well as to computational mechanics problems.

1. Ability to develop and use reduced order models for engineering applications.
2. Ability to computationally implement the different ROM techniques.
3. Ability to select the most appropriate technique for each problem.

Singular Value Decomposition (SVD), Modal Analysis, Proper Orthogonal Decomposition, Hyperreduction, Proper Generalized Decomposition, Reduced bases, Response surfaces.

STUDY LOAD

<table>
<thead>
<tr>
<th>Type</th>
<th>Hours</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours large group</td>
<td>15,0</td>
<td>12.00</td>
</tr>
<tr>
<td>Hours medium group</td>
<td>15,0</td>
<td>12.00</td>
</tr>
<tr>
<td>Hours small group</td>
<td>7,5</td>
<td>6.00</td>
</tr>
<tr>
<td>Guided activities</td>
<td>7,5</td>
<td>6.00</td>
</tr>
<tr>
<td>Self study</td>
<td>80,0</td>
<td>64.00</td>
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</tbody>
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Total learning time: 125 h

CONTENTS

Math preliminaries

Description:
Vector and matrix norms (useful when defining the optimization objectives), rank, column space, span of the column. Eigenvalues and Eigenvectors. Types of matrices and their properties (symmetry, orthogonality, orthonormality).

Mathematical Preliminaries: practical session

Specific objectives:
This section will serve as a brush up on the necessary Linear Algebra concepts required for the understanding of model reduction when motivated as optimization problems.

Full-or-part-time: 7h 11m
Theory classes: 1h
Practical classes: 2h
Self study : 4h 11m
Data and SVD

Description:
This section will introduce different types of data and its representation in matrix form. The data will be treated in a broad sense: experiments, images, numerical simulations etc. The singular value decomposition (SVD) will be introduced as a means for finding the most relevant intrinsic patterns in these data.

Specific objectives:
Familiarize the students with the application of SVD using programming environment.

Full-or-part-time: 7h 11m
Theory classes: 1h
Practical classes: 2h
Self study: 4h 11m

POD ("proper orthogonal decomposition")

Description:
Introduction to POD: SVD applied to PDEs. Dimensionality reduction as a means of representing spatio-temporal behavior of complex systems using low-dimensional approximations ("low-dimensional patterns of dynamic activity").

POD basics: PDEs, separation of variables. Representation of the spatial distribution of the variable using modal basis. Galerkin projection using orthonormal basis functions. Ways of selecting the basis.

Application of POD to FEM model.

Full-or-part-time: 10h 48m
Theory classes: 3h
Practical classes: 1h 30m
Self study: 6h 18m

Modal analysis

Description:
Modal Analysis: an "a priori" alternative to POD. Differences and similarities. Application to modeling of a linear structure.

Modal analysis: practical session
An alternative approach to ROM is the a priori approach, with no need of a prior sampling the parametric space and a posterior solving of the reduced equation. The idea of the PGD is based on considering parameters as extra coordinates and handling the high-dimensionality by using separated approximations.

Full-or-part-time: 14h 23m
Theory classes: 4h
Practical classes: 2h
Self study: 8h 23m
## Hyperreduction

**Description:**
Hyperreduction: a means for dimensionality reduction of non-linear problems.

**Full-or-part-time:** 3h 35m  
Practical classes: 1h 30m  
Self study: 2h 05m

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## Assignments presentation

**Full-or-part-time:** 18h  
Laboratory classes: 7h 30m  
Self study: 10h 30m

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## Parametric problems

**Description:**
The lecture aims at presenting the parametric problem (full order) to be used as a demonstrator for all the concepts introduced in the second part of the course. The set of parametric solutions is analyzed as a low-order manifold in a high-dimension Euclidean space. Different techniques to describe the set of solutions are quickly reviewed.

**Full-or-part-time:** 7h 11m  
Theory classes: 3h  
Self study: 4h 11m

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## Reduced basis

**Description:**
The full-order model is transformed into a reduced-order model using a low-dimensional representation computed offline. Attention is paid to different constructions of the reduced bases, and their comparison. Either following a greedy technique (adding a new term only if it is non-redundant) or orthogonalizing the basis.

Reduced bases and a posteriori ROM: practical session

**Full-or-part-time:** 7h 11m  
Theory classes: 1h 30m  
Practical classes: 1h 30m  
Self study: 4h 11m

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## PGD

**Description:**
The PGD philosophy goes far beyond the solution of the parametric problem and allows building explicit surrogates for QoI described as simple operations of the solution. The lecture aims at producing the surrogates corresponding to different QoI using the available PGD operations.

**Full-or-part-time:** 7h 11m  
Practical classes: 3h  
Self study: 4h 11m
Response surfaces

Description:
The simplest approach to build a surrogate model for any parametric QoI is reconstructing the response function as a multiparametric field. Classical interpolation and functional fitting techniques are reviewed, paying special attention to the offline sampling. The sensitivity of the obtained solutions to the choice of location of the sampling points is also analyzed.

Full-or-part-time: 7h 11m
Theory classes: 1h 30m
Practical classes: 1h 30m
Self study : 4h 11m

GRADING SYSTEM

The grade for the course is obtained in the continuous assessment mode.

Continuous assessment consists of doing different activities ("assignments") of didactic nature, carried out during the course (in the classroom and at home).

Additionally, two tests (examinations) will be introduced: 1 midterm and one final term. It consists of a part with questions about theoretical concepts associated with the learning objectives of the subject, and/or a practical part related to the exercises and assignments.

BIBLIOGRAPHY

Basic: