Course guides
34957 - GT - Graph Theory

Last modified: 31/05/2020

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2020 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: MARCOS NOY SERRANO

Others: Primer quadrimestre:
MARCOS NOY SERRANO - A
ORIOL SERRA ALBO - A

PRIOR SKILLS

Elementary Calculus and Linear Algebra; basic notions and abilities in combinatorics and probability.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Sessions of presentation of material alternate with sessions with student presentations of problems and specific topics. The active participation of students is a requirement for the evaluation of the course.

LEARNING OBJECTIVES OF THE SUBJECT

Extremal graph theory
Application of spectral techniques to the study of graphs.
Application of the probabilsitic method.
Properties of almost all graphs.
Properties of Cayley and vertex symmetric graphs.
Graphs on surfaces.
Minors.

STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

CONTENTS

Spectral techniques in Graph Theory

Description:
Adjacency and Laplacian matrix. Spectral properties. Cospectral graphs. Graph invariants and spectral properties: chromatic number, Cheeger constant, expansion properties, maxcut, bisection width. The matrix tree theorem. Random walks in graphs. Shannon capacity.

Specific objectives:
Computation of spectra. Circulant graphs. Spectra and graph operations. Obtention of spectral bounds for graph invariants.

Full-or-part-time: 12h
Theory classes: 12h

Symmetries in graphs

Description:
Vertex symmetric and Edge symmetric graphs. Cayley graphs. Highly symmetric graphs

Specific objectives:
Circulant graphs. Hypercubes. Toroidal graphs.

Full-or-part-time: 1h
Theory classes: 1h

Minors and treewidth

Description:
Minors. Minor closed classes. Well quasi-ordering. Graph minor theorem for trees. Tree decomposition. Tree width.

Specific objectives:
Classes defined by forbidden minors. Serie-Parallel graphs. k-trees and tree width.

Full-or-part-time: 11h
Theory classes: 11h

Graphs on surfaces

Description:
Planar graphs. Kuratowski theorem. Triangulations. Graphs on surfaces. Genus.

Specific objectives:
Euler formula. Planar separator theorem

Full-or-part-time: 4h
Theory classes: 4h

Graph homomorphisms

Description:
Graph homomorphisms. Retracts and Cores. The homomorphism order. Antichains.

Specific objectives:
Homomorphisms and colorings. Fractional and circular chromatic numbers.

Full-or-part-time: 6h
Theory classes: 6h

Random graphs

Description:
Erdos-Rényi model of random graphs. Probabilistic method. Properties of almost all graphs. Threshold functions. Evolution of random graphs.

Specific objectives:
Graphs with large girth and large chromatic number. Expansion properties of random graphs. Threshold for connectivity. The Poisson paradigm.

Full-or-part-time: 12h
Theory classes: 12h

Extremal Graph Theory

Description:
Extremal problems in graph theory. Turán theorem. The Erdos-Stone-Simonovits theorem. Stability of extremal graphs. Szemerédi regularity lemma.

Specific objectives:
Counting Lemma and Removal Lemma.Applications of Szemerédi regularity Lemma.

Full-or-part-time: 12h
Theory classes: 12h

GRADING SYSTEM

The evaluation of the course is based on the weekly work on problems proposed in the presentation sessions. There will be a final comprehensive exam based on the problem sessions during the course.

EXAMINATION RULES.

The active participation in the course is a requirement for the evaluation of the final exam.

BIBLIOGRAPHY

Basic:
- Alon, Noga; Spencer Joel. The Probabilistic Method. 2016. Wiley,
- Brouwer, A. E; Haemers, W. H. Spectra of Graphs. New York [etc.]: Springer, cop. 2012. ISBN 978-1-4614-1938-9.
- Diestel, Reinhard. Graph theory. 3rd ed. Berlin: Springer, 2005. ISBN 3540261826.
- Hell, Pavol; Nesetril, Jaroslav. Graphs and homomorphisms. Oxford: Oxford University Press, 2004. ISBN 0198528175.
- Frieze, Alan; Karonski, Michal. Introduction to random graphs. Cambridge, [etc.]: Cambridge University Press, cop. 2016. ISBN 978-1107118508.

Complementary:
- Biggs, Norman L. Algebraic graph theory. 2nd ed. New York: Cambridge University Press, 1993. ISBN 0521458978.
- Chung, Fan R. K. Spectral Graph Theory. Providence: American Mathematical Society, 1997. ISBN 0821803158.