Course guides
34963 - ACPDE - Advanced Course in Partial Differential Equations

Last modified: 09/06/2020

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.
981 - CRM - Mathematical Research Centre.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2020 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: ALBERT MAS BLESA

Others: Segon quadrimestre:
XAVIER CABRE VILAGUT - A
JUAN CARLOS FELIPE NAVARRO - A
ALBERT MAS BLESA - A

PRIOR SKILLS

Basic knowledge of Partial Differential Equations (undergraduate level).
Basic knowledge of Mathematical Analysis (undergraduate level).

REQUIREMENTS

Undergraduate courses in Partial Differential Equations and in Mathematical Analysis.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Classes will combine theoretical aspects and proofs with resolution of concrete problems and exercises.
Further reading from the bibliography will be given often.

LEARNING OBJECTIVES OF THE SUBJECT

This course is intended to be an introduction to modern methods for solving elliptic partial differential equations. However, some insights to classical solutions to parabolic and hyperbolic equations will also be given. The objectives of the course are:
- understand the classical methods to solve the transport, wave, heat, Laplace, and Poisson equations,
- understand the role of Sobolev norms and compact embeddings to solve PDEs and find spectral decompositions,
- learn the modern methods to solve elliptic PDEs.

STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

CONTENTS

Classical methods in PDEs

Description:
[This topic will only be treated in the exercises sessions.] Classical solutions to the transport, wave, heat, Laplace, and Poisson equations. Maximum principles, Green's functions, separation of variables, energy methods, probabilistic interpretation.

Full-or-part-time: 46h
Theory classes: 15h
Self study : 31h

Hilbert space techniques

Description:
Orthogonal projections, Riesz-Fréchet representation theorem, Lax-Milgram theorem.

Full-or-part-time: 25h
Theory classes: 8h
Self study : 17h

Sobolev spaces

Description:
Mollifiers, Fréchet-Kolmogorov theorem, distributions, Sobolev norms, Poincaré inequality, compact embeddings, approximation by smooth functions, traces.

Full-or-part-time: 29h
Theory classes: 9h
Self study : 20h

Weak formulation and the weak maximum principle

Description:
Weak solutions via Hilbert space techniques and interpretation, comparison principles in the weak formulation.

Full-or-part-time: 25h
Theory classes: 8h
Self study : 17h

Regularity theory

Description:
Boundedness of weak solutions, Sobolev-Gagliardo-Nirenberg inequality, regularity in Sobolev spaces, the translation method, bootstrap technique.

Full-or-part-time: 25h 30m
Theory classes: 8h
Self study : 17h 30m

Eigenvalues

Description:
Spectral decompositions, applications to (time dependent) evolution equations, Rayleigh quotient, description of the first eigenvalue for the Dirichlet problem on a bounded domain.

Full-or-part-time: 17h
Theory classes: 6h
Self study : 11h

Nonlinear problems

Description:
Calculus of variations, monotone iteration method, obstacle problems.

Full-or-part-time: 20h
Theory classes: 6h
Self study : 14h

GRADING SYSTEM

The evaluation of the course is based on:
- the resolution of problems proposed in class (40%),
- a midterm exam (20%),
- a final comprehensive exam (40%).
The active participation during the course will be a requirement for the evaluation of the final exam.

BIBLIOGRAPHY

Basic:
- Chipot, M. Elliptic Equations: An Introductory Course [on line]. Basel: Birkhäuser Basel, 2009 [Consultation: 26/06/2020]. Available on: http://dx.doi.org/10.1007/978-3-7643-9982-5. ISBN 9783764399825.
- Evans, Lawrence C. Partial differential equations. Providence, Rhode Island: American Mathematical Society, 1998. ISBN 0821807722.
- Salsa, Sandro. Partial differential equations in action : from modelling to theory [on line]. Milan: Springer, 2008 [Consultation: 19/05/2020]. Available on: https://ebookcentral.proquest.com/lib/upcatalunya-ebooks/detail.action?docID=3062992. ISBN 9788847007512.
- Brézis, H. Análisis funcional : teoría y aplicaciones. Madrid: Alianza, 1984. ISBN 8420680885.

Complementary:
- Folland, G. B. Introduction to partial differential equations. 2nd ed. Princeton (N.J.): Princeton University Press, 1995. ISBN 0691043612.
- Gilbarg, David; Trudinger, Neil S. Elliptic partial differential equations of second order. 2nd ed., rev. third printing. Berlin: Springer-Verlag, 1998. ISBN 354013025X.
- Necas, Jindrich. Introduction to the theory of nonlinear elliptic equations. Chichester: John Wiley & Sons, 1986. ISBN 0471908940.
- Struwe, Michael. Variational methods : applications to nonlinear partial differential equations and hamiltonian systems [on line]. 2nd rev. and substantially expanded ed. Berlin: Springer, 1996 [Consultation: 19/05/2020]. Available on: http://dx.doi.org/10.1007/978-3-540-74013-1. ISBN 3540520228.