Course guide
34966 - VD - Differentiable Manifolds

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.
Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).
(Optional subject).
Academic year: 2022  ECTS Credits: 7.5  Languages: English

LECTURER

Coordinating lecturer: EVA MIRANDA GALCERÁN
Others: Segon quadrimestre:
PAU MIR GARCÍA - A
EVA MIRANDA GALCERÁN - A

PRIOR SKILLS

Basic courses on algebra, calculus, topology and differential equations, and calculus on manifolds. Students from the FME are supposed to have taken "Varietats Diferenciables" (optional 4th year course).

This is not a basic course and the students are assumed to have attended previous courses on differential geometry and smooth manifolds. Students feeling that they may not fulfill the requisites are invited to discuss their case with the lecturers. It is totally possible for prospective students with less knowledge in these topics to follow this course provided they are willing to make up for the gap with individual work during the course and/or by reading some recommended bibliography prior to the beginning of the course.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Theory classes and tutorial sessions will be used to present and develop the contents of the course. Along the course the students will be given problems to solve as homework.
LEARNING OBJECTIVES OF THE SUBJECT

The subject focuses on some of the fundamental topics of differential geometry and its applications to different areas including mathematical physics and Dynamical systems.

By the end of the course, students should be able to:
- understand all the ideas developed along the course.
- apply the studied concepts to other areas of pure mathematics, physics and engineering.
- integrate in a research group on these kinds of topics and their applications.
- search and understand the scientific literature on the subject.
- write and present an essay on mathematics.

STUDY LOAD

<table>
<thead>
<tr>
<th>Type</th>
<th>Hours</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours large group</td>
<td>60,0</td>
<td>32.00</td>
</tr>
<tr>
<td>Self study</td>
<td>127,5</td>
<td>68.00</td>
</tr>
</tbody>
</table>

**Total learning time:** 187.5 h

CONTENTS

**Complements in Differential Geometry**

**Description:**
Brief survey of manifold theory and differential geometry including differential forms.
We also plan to talk about differentiable distributions and study its integration via the theorem of Frobenius. This will lead us to introducing several examples of foliations.

**Full-or-part-time:** 14h 52m
Theory classes: 6h
Self study : 8h 52m

**Introduction to Differential Topology**

**Description:**
We present a brief introduction to the theory of Differential Topology which includes basic notions in transversality, singularity theory and Morse theory.

**Full-or-part-time:** 14h 40m
Theory classes: 8h
Self study : 6h 40m

**Introduction to Lie theory**

**Description:**
A Lie group is a group endowed with a smooth manifold structure which is compatible with the group operation. In this chapter we provide an introduction to the main aspects of the theory of Lie groups and Lie algebras taking matrix Lie groups as starting point.

**Full-or-part-time:** 16h 20m
Theory classes: 8h
Self study : 8h 20m


**Lie group actions on smooth manifolds**

**Description:**
We study Lie group actions on smooth manifolds and relate both geometries via the notions of isotropy group and orbit.

**Full-or-part-time:** 18h
- Theory classes: 4h
- Theory classes: 4h
- Self study: 5h
- Self study: 5h

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**Basic notions on De Rham Cohomology**

**Description:**
We define De Rham cohomology and compare it to other cohomologies. (Depending on the preliminary knowledge of the students, this chapter may be considered as an APPENDIX)

**Full-or-part-time:** 8h
- Theory classes: 3h
- Self study: 5h

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**Introduction to Symplectic and Poisson Geometry**

**Description:**
We provide a comprehensive introduction to symplectic and Poisson manifolds with special focus on examples. Starting with symplectic manifolds, we will explain Moser’s trick and some applications to normal form theorems such as the Darboux theorem and the classification of symplectic surfaces. We introduce the notion of Hamiltonian vector field, symplectic vector field and Hamiltonian System. Special attention will be given to examples provided by the realm of integrable systems. In particular the action-angle theorem of Arnold-Liouville will be presented and the notion of moment map and Hamiltonian group action. We end the chapter introducing the basic concepts in Poisson geometry (a natural generalization of Symplectic geometry) and proving a decomposition theorem (Weinstein's splitting theorem) in terms of a symplectic leaf of the symplectic foliation.

**Full-or-part-time:** 31h 40m
- Theory classes: 15h
- Self study: 16h 40m

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**GRADING SYSTEM**

There will be exam(s) which will contribute to the final grade in a 40% and an essay that will contribute to the final grade in another 40%. Students would choose, together with the lecturers, a topic that complements or advances the material taught during the course, according to their mathematical interests. The remaining 20% is reserved to problem solving which can be evaluated by simply solving some assignments of the regular list of problems.

**EXAMINATION RULES.**

The final grade awarded to the student would we computed as follows:

40% exam(s) + 40% essay + 20% problem solving

The grade "exam(s)" includes the one of final exam but may also include other examination material such as ATENEA questionnaires or take-home exercises. The choices and number of exams will depend on several factors including the ratio presentential versus online teaching.
BIBLIOGRAPHY

Basic:
- Eva Miranda, Pau Mir and Cédric Oms. Notes of the course.

Complementary:

RESOURCES

Other resources:
Notes on the Geometry and Dynamics of singular symplectic manifolds (notes on the FSMP course by Eva Miranda)

Course on youtube by Professor Eva Miranda on Lie group actions
https://www.youtube.com/channel/UC8Fzyf58s0EiZ-gdYg2ghw/view_as=subscriber
Course on youtube by Professor Eva Miranda on Symplectic and Poisson Geometry
https://www.youtube.com/channel/UC8Fzyf58s0EiZ-gdYg2ghw/view_as=subscriber