Course guides
34966 - VD - Differentiable Manifolds

Last modified: 17/06/2020

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2020 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: EVA MIRANDA GALCERÁN

Others: Segon quadrimestre:
EVA MIRANDA GALCERÁN - A
CEDRIC OMS - A

PRIOR SKILLS

Basic courses on algebra, calculus, topology and differential equations, and calculus on manifolds. Students from the FME are supposed to have taken "Varietats Diferenciables" (optional 4th year course).

This is not a basic course and the students are assumed to have attended previous courses on differential geometry and smooth manifolds. Students feeling that they may not fulfill the requisites are invited to discuss their case with the lecturers. It is totally possible for prospective students with less knowledge in these topics to follow this course provided they are willing to make up for the gap with individual work during the course and/or by reading some recommended bibliography prior to the beginning of the course.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Theory classes and tutorial sessions will be used to present and develop the contents of the course. Along the course the students will be given problems to solve as homework.

LEARNING OBJECTIVES OF THE SUBJECT

The subject focuses on some of the fundamental topics of differential geometry and its applications to different areas including mathematical physics and Dynamical systems.

By the end of the course, students should be able to:
- understand all the ideas developed along the course.
- apply the studied concepts to other areas of pure mathematics, physics and engineering.
- integrate in a research group on these kinds of topics and their applications.
- search and understand the scientific literature on the subject.
- write and present an essay on mathematics.

STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

CONTENTS

Complements in Differential Geometry

Description:
Brief survey of manifold theory and differential geometry including differential forms.
We also plan to talk about differentiable distributions and study its integration via the theorem of Frobenius. This will lead us to introducing several examples of foliations.

Full-or-part-time: 14h 52m
Theory classes: 6h
Self study : 8h 52m

Introduction to Differential Topology

Description:
We present a brief introduction to the theory of Differential Topology which includes basic notions in transversality, singularity theory and Morse theory.

Full-or-part-time: 14h 40m
Theory classes: 8h
Self study : 6h 40m

Introduction to Lie theory

Description:
A Lie group is a group endowed with a smooth manifold structure which is compatible with the group operation. In this chapter we provide an
introduction to the main aspects of the theory of Lie groups and Lie algebras taking matrix Lie groups as starting point.

Full-or-part-time: 16h 20m
Theory classes: 8h
Self study : 8h 20m

Lie group actions on smooth manifolds

Description:
We study Lie group actions on smooth manifolds and relate both geometries via the notions of isotropy group and orbit.

Full-or-part-time: 18h
Theory classes: 4h
Theory classes: 4h
Self study : 5h
Self study : 5h

Basic notions on De Rham Cohomology

Description:
We define De Rham cohomology and compare it to other cohomologies. (Depending on the preliminary knowledge of the students, this chapter may be considered as an APPENDIX)

Full-or-part-time: 8h
Theory classes: 3h
Self study : 5h

Introduction to Symplectic and Poisson Geometry

Description:
We provide a comprehensive introduction to symplectic and Poisson manifolds with special focus on examples. Starting with symplectic manifolds, we will explain Moser's trick and some applications to normal form theorems such as the Darboux theorem and the classification of symplectic surfaces. We introduce the notion of Hamiltonian vector field, symplectic vector field and Hamiltonian System. Special attention will be given to examples provided by the realm of integrable systems. In particular the action-angle theorem of Arnold-Liouville will be presented and the notion of moment map and Hamiltonian group action. We end the chapter introducing the basic concepts in Poisson geometry (a natural generalization of Symplectic geometry) and proving a decomposition theorem (Weinstein's splitting theorem) in terms of a symplectic leaf of the symplectic foliation.

Full-or-part-time: 31h 40m
Theory classes: 15h
Self study : 16h 40m

GRADING SYSTEM

There will be exam(s) which will contribute to the final grade in a 50% and an essay that will contribute to the final grade in another 50%. Students would choose, together with the lecturers, a topic that complements or advances the material taught during the course, according to their mathematical interests.

EXAMINATION RULES.

The final grade awarded to the student would we computed as follows:

50% exam(s) + 50% essay

The grade "exam(s)" includes the one of final exam but may also include other examination material such as ATENEA questionnaires or take-home exercises. The choices and number
of exams will depend on several factors including the ratio presential versus online teaching.

BIBLIOGRAPHY

Basic:
- Lee, John M. Introduction to smooth manifolds [on line]. New York: Springer-Verlag, 2003 [Consultation: 19/05/2020]. Available on: https://doi.org/10.1007/978-1-4419-9982-5. ISBN 0387954481.
- Duistermaat, J. J. ; Kolk, Johan A. C. Lie groups. Berlin: Springer-Verlag, 2000. ISBN 3540152938.
- Guillemin, Victor; Pollack, Alan. Differential topology. Reprint of the 1974 original. AMS Chelsea Publishing,
- Bröcker, T.; Jänich, K. Introduction to differential topology. Cambridge University Press, 1982. ISBN 978-0521284707.
- Fegan, H.D.. Introduction to compact lie groups. World Scientific, 1991. ISBN 9810236867.
- Eva Miranda, Pau Mir and Cédric Oms. Notes of the course.
- Knauf, A.. Mathematical Physics: Classical Mechanics. Springer, 2018. ISBN 978-3-662-55774-7.
- Guillemin, V; Sternberg, Shlomo. Symplectic techniques in physics. Repr. with corrections. Cambridge [etc.]: Cambridge University Press, 1990. ISBN 978-0521389907.
- Milnor, John W. Topology from the differentiable viewpoint. Rev. ed. Princenton: Princeton University Press, 1997. ISBN 978-0691048338.
- Munkres, J.R. Elementary differential topology. ISBN 978-0691090931.
- Warner, Frank W. Foundations of differentiable manifolds and Lie groups. New York [etc.]: Springer, cop. 1983. ISBN 9780387908946.
- Tu, Loring W. An Introduction to manifolds [on line]. 2nd ed. New York: Springer, cop. 2011 [Consultation: 19/05/2020]. Available on: https://doi.org/10.1007/978-1-4419-7400-6. ISBN 9781441973993.
- Cannas da Silva, Anna. Lectures on symplectic geometry [on line]. Springer-Verlag, 2008 [Consultation: 19/05/2020]. Available on: https://link.springer.com/book/10.1007/978-3-540-45330-7.

Complementary:
- Bott, Raoul; Tu, Loring W. Differential forms in algebraic topology. New York: Springer-Verlag, 1982. ISBN 0387906134.
- Audin, Michèle; Damian, Mihai. Morse theory and Floer homology. New York: Springer, [2014]. ISBN 978-1447154952.
- Gallier, Jean;Quaintance, Jocelyn. Notes on differential geometry and Lie groups [on line]. University of Pennsylvania, 2016 [Consultation: 19/05/2020]. Available on: http://www.cis.upenn.edu/~jean/gbooks/manif.html.
- Nakahara, Mikio. Geometry, topology, and physics. 2nd ed. New York [etc.]: Taylor & Francis, cop. 2003. ISBN 0750306068.
- Audin, Michèle. Torus actions on symplectic manifolds. 2nd ed. Birkhäuser, 2004.
- Warner, Frank W. Foundations of differentiable manifolds and lie groups. New York, NY [etc.]: Springer-Verlag, cop. 1971. ISBN 0387908943.
- Olver, Peter J. Applications of Lie groups to differential equations. New York: Springer-Verlag, 1986. ISBN 0387940073.

RESOURCES

Other resources:
Notes on the Geometry and Dynamics of singular symplectic manifolds (notes on the FSMP course by Eva Miranda)

Course on youtube by Professor Eva Miranda on Lie group actions
https://www.youtube.com/channel/UC8Fzyf58s0EiZ-gdYgz2ghw?view_as=subscriber
Course on youtube by Professor Eva Miranda on Symplectic and Poisson Geometry
https://www.youtube.com/channel/UC8Fzyf58s0EiZ-gdYgz2ghw?view_as=subscriber