34966 - VD - Differentiable Manifolds

Coordinating unit: 200 - FME - School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2018
Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Teaching unit Optional)
ECTS credits: 7,5
Teaching languages: English

Teaching staff
Coordinator: EVA MIRANDA GALCERÁN
Others: Segon quadrimestre:
EVA MIRANDA GALCERÁN - A
CEDRIC OMS - A

Prior skills
Basic courses on algebra, calculus, topology and differential equations, and calculus on manifolds. Students from the FME are supposed to have taken "Varietats Diferenciables" (optional 4th year course).

This is not a basic course and the students are assumed to have attended previous courses on differential geometry and smooth manifolds. Students feeling that they may not fulfill the requisites are invited to discuss their case with the lecturers. It is totally possible for prospective students with less knowledge in these topics to follow this course provided they are willing to make up for the gap with individual work during the course and/or by reading some recommended bibliography prior to the beginning of the course.

Degree competences to which the subject contributes

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.
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Teaching methodology

Theory classes and tutorial sessions will be used to present and develop the contents of the course. Along the course the students will be given problems to solve as homework.

Learning objectives of the subject

The subject focuses on some of the fundamental topics of differential geometry and its applications to different areas including mathematical physics and Dynamical systems.

By the end of the course, students should be able to:
- understand all the ideas developed along the course.
- apply the studied concepts to other areas of pure mathematics, physics and engineering.
- integrate in a research group on these kinds of topics and their applications.
- search and understand the scientific literature on the subject.
- write and present an essay on mathematics.

Study load

<p>| Total learning time: 187h 30m | Hours large group: | 60h | 32.00% |
| Self study: | 127h 30m | 68.00% |</p>
<table>
<thead>
<tr>
<th>Content</th>
<th>Learning time: 14h 52m</th>
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</thead>
<tbody>
<tr>
<td>Complements in Differential Geometry</td>
<td></td>
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<tr>
<td><strong>Description:</strong></td>
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<tr>
<td>Brief survey of manifold theory and differential geometry including differential forms.</td>
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<tr>
<td>We also plan to talk about differentiable distributions and study its integration via the theorem of Frobenius. This will lead us to introducing several examples of foliations.</td>
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<tr>
<td>Introduction to Differential Topology</td>
<td>Learning time: 14h 40m</td>
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<tr>
<td><strong>Description:</strong></td>
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<tr>
<td>We present a brief introduction to the theory of Differential Topology which includes basic notions in transversality, singularity theory and Morse theory.</td>
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<tr>
<td>Introduction to Lie theory</td>
<td>Learning time: 16h 20m</td>
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<td><strong>Description:</strong></td>
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<tr>
<td>A Lie group is a group endowed with a smooth manifold structure which is compatible with the group operation. In this chapter we provide an introduction to the main aspects of the theory of Lie groups and Lie algebras taking matrix Lie groups as starting point.</td>
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<tr>
<td>Lie group actions on smooth manifolds</td>
<td>Learning time: 9h</td>
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<td><strong>Description:</strong></td>
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<tr>
<td>We study Lie group actions on smooth manifolds and relate both geometries via the notions of isotropy group and orbit.</td>
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<table>
<thead>
<tr>
<th>Basic notions on De Rham Cohomology</th>
<th>Learning time: 8h</th>
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<tbody>
<tr>
<td></td>
<td>Theory classes: 3h</td>
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<td>Self study: 5h</td>
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**Description:**
We define De Rham cohomology and compare it to other cohomologies.

<table>
<thead>
<tr>
<th>Introduction to Symplectic and Poisson Geometry</th>
<th>Learning time: 31h 40m</th>
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<tbody>
<tr>
<td></td>
<td>Theory classes: 15h</td>
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<tr>
<td></td>
<td>Self study: 16h 40m</td>
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</tbody>
</table>

**Description:**
We provide a comprehensive introduction to symplectic and Poisson manifolds with special focus on examples. Starting with symplectic manifolds, we will explain Moser's trick and some applications to normal form theorems such as the Darboux theorem and the classification of symplectic surfaces. We introduce the notion of Hamiltonian vector field, symplectic vector field and Hamiltonian System. Special attention will be given to examples provided by the realm of integrable systems. In particular the action-angle theorem of Arnold-Liouville will be presented and the notion of moment map and Hamiltonian group action. We end the chapter introducing the basic concepts in Poisson geometry (a natural generalization of Symplectic geometry) and proving a decomposition theorem (Weinstein's splitting theorem) in terms of a symplectic leaf of the symplectic foliation.

### Qualification system

There will be a final exam, as well as the possibility to write an optional essay that would contribute to the final grade. Students would choose, together with the lecturers, a topic that complements or advances the material taught during the course, according to their mathematical interests.

### Regulations for carrying out activities

The final grade awarded to the student would be computed as follows:

- **Case A:** an student that does only the final exam. Then the final grade would be that of the exam.
- **Case B:** an student that does the final exam AND submits a written essay. Then the final note would be the result of\[ \text{MAX}(\text{exam}, 35\% \times \text{exam} + 40\% \times \text{essay} + 25\% \times \text{exercises}) \]
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**Bibliography**

**Basic:**
- Fegan, H. D. Introduction to compact Lie groups. ISBN 9810236867.

**Complementary:**

**Others resources:**