



## Course guide

### 820007 - CAL - Calculus

Last modified: 27/10/2022

**Unit in charge:** Barcelona East School of Engineering  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** BACHELOR'S DEGREE IN BIOMEDICAL ENGINEERING (Syllabus 2009). (Compulsory subject).  
BACHELOR'S DEGREE IN CHEMICAL ENGINEERING (Syllabus 2009). (Compulsory subject).  
BACHELOR'S DEGREE IN ELECTRICAL ENGINEERING (Syllabus 2009). (Compulsory subject).  
BACHELOR'S DEGREE IN ENERGY ENGINEERING (Syllabus 2009). (Compulsory subject).  
BACHELOR'S DEGREE IN INDUSTRIAL ELECTRONICS AND AUTOMATIC CONTROL ENGINEERING (Syllabus 2009). (Compulsory subject).  
BACHELOR'S DEGREE IN MECHANICAL ENGINEERING (Syllabus 2009). (Compulsory subject).  
BACHELOR'S DEGREE IN MATERIALS ENGINEERING (Syllabus 2010). (Compulsory subject).

**Academic year:** 2022    **ECTS Credits:** 6.0    **Languages:** Catalan, Spanish, English

#### LECTURER

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**Coordinating lecturer:** ANGELES CARMONA MEJIAS - MAGDA LILIANA RUIZ ORDOÑEZ

**Others:** Primer quadrimestre:  
ENRIC AMADO VICENTE - Grup: T21, Grup: T22, Grup: T23  
ANGELES CARMONA MEJIAS - Grup: M51, Grup: M52  
MERCÉ CLAVEROL AGUAS - Grup: M21, Grup: M22, Grup: M42, Grup: M52, Grup: M81, Grup: M82, Grup: M91  
ADRIÀ DOMINGO GIMENEZ - Grup: X22  
RAIMON ELGUETA MONTO - Grup: X11, Grup: X12, Grup: X22  
ANDRES MARCOS ENCINAS BACHILLER - Grup: M71, Grup: M72  
ALFONSO ESCOBOSA FERNANDEZ - Grup: T11, Grup: T12  
MARIONA GONZÁLEZ ESTEVE - Grup: T12, Grup: T22, Grup: X11, Grup: X21  
SERGIO GONZÁLEZ LÓPEZ - Grup: M31, Grup: M41, Grup: M42, Grup: M72, Grup: X21, Grup: X22  
NILS GUTIÉRREZ VON PORAT - Grup: M81  
PERE LOPEZ BROSA - Grup: M61  
ALBERT MAS BLESÀ - Grup: M41, Grup: M42, Grup: X21, Grup: X22  
NURIA PARES MARINE - Grup: M11, Grup: M32, Grup: M71  
FRANCESC POZO MONTERO - Grup: M11  
JOAN QUINTANA COMPTE - Grup: M41, Grup: M51, Grup: M62, Grup: M91, Grup: M92  
MAGDA LILIANA RUIZ ORDOÑEZ - Grup: M21, Grup: M61, Grup: M62, Grup: M92  
ROGER VALDÉS I MARTÍN - Grup: M22, Grup: M31, Grup: M32

#### PRIOR SKILLS

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This course requires no previous skills.

#### REQUIREMENTS

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This course has no prerequisites.



## DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

### Specific:

2. Solve mathematical problems that may arise in engineering. Apply knowledge of linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and partial differential equations; numerical methods; numerical algorithms; statistics and optimisation.

### Transversal:

1. SELF-DIRECTED LEARNING - Level 1. Completing set tasks within established deadlines. Working with recommended information sources according to the guidelines set by lecturers.

## TEACHING METHODOLOGY

The course uses the expositive methodology by 40% and individual work by 60%.

## LEARNING OBJECTIVES OF THE SUBJECT

General objectives: Students will learn the fundamental concepts of single variable calculus, developing the capacity of abstraction and applying these techniques to mathematical problems encountered in engineering.

## STUDY LOAD

Type	Hours	Percentage
Hours small group	15,0	10.00
Hours large group	45,0	30.00
Self study	90,0	60.00

**Total learning time:** 150 h

## CONTENTS

### Sets of numbers

#### Description:

- The set of real numbers: Supremum axiom.
- The set of complex numbers: Binomial, polar, and exponential form. Operations with complex numbers. Powers and roots. Euler's formula. Definition of hyperbolic functions and their relation to trigonometry and complex numbers.

#### Specific objectives:

- The students will learn:
- Supremum axiom, key to understand the completeness of real numbers.
  - to operate with complex numbers.
  - to establish relationships between binomial, polar, and exponentials forms.

#### Related activities:

- Lab session 1. Conics
- Lab session 2. Complex numbers

#### Full-or-part-time: 30h

- Theory classes: 8h
- Laboratory classes: 4h
- Self study : 18h



### Functions of real variable. Limits and continuity.

**Description:**

- Concept of function. Domain and codomain. Basic functions in engineering: Heaviside function. Operating with functions. Composition of functions. Inverse function.
- Limit of a function in a point. Definition and basic properties. One-sided limits. Evaluating limits. Extending the concept of a limit (infinite limits, limits at infinity). Limits of indeterminate forms. Infinities and equivalent infinities. Orders of infinity.
- Continuity. Continuity theorems (Weierstrass, Bolzano, intermediate value theorem).

**Specific objectives:**

Students will learn:

- to represent a real-valued function.
- to understand the importance of the concept of limit and its relationship to continuity.

**Related activities:**

Lab Session 3. Limits and continuity

**Full-or-part-time:** 30h

Theory classes: 10h

Laboratory classes: 2h

Self study : 18h

### Differentiation of real-valued functions

**Description:**

- Derivative of a function at a point. Relationship between differentiability and continuity. The derivative as a function. Geometric interpretation: Tangent line. The chain rule. Implicit derivative. Calculus of derivatives.
- Local approximation of a function. Taylor polynomials. Error formula.
- Mean value theorems (Rolle, Cauchy, Lagrange).
- Extrema of a function in an interval.

**Specific objectives:**

The student will learn:

- the basic concepts of differentiation.
- to understand the geometric interpretation of the derivative and its applications in engineering.
- to master and apply the elementary properties of the differentiable functions.
- to master the computation of derivatives, both analytically and with the help of mathematical software.
- to model and solve several problems by computing derivatives: optimization, approximation of functions, and qualitative study of functions.

**Related activities:**

Lab session 6. PART I: Taylor polynomial

**Full-or-part-time:** 35h

Theory classes: 12h

Laboratory classes: 2h

Self study : 21h



## Integration of real-value functions

### Description:

- Primitive functions.
- Integration methods: direct methods, change of variable, integration by parts, trigonometric integrals.
- Definite integral (Riemann integral). Integrable functions. The fundamental theorem of calculus. Barrow's rule.
- Computation of areas of plane regions. Applications.
- Improper integrals.

### Specific objectives:

Students will learn:

- to express in terms of integrals the problem of computing the area of a plane region.
- to understand the relationship between derivatives and integrals, given by the fundamental theorem of calculus.
- to use the Barrow's rule.
- to compute some improper integrals of continuous functions on an unbounded interval, and improper integrals of functions with a singularity inside a bounded interval.

### Related activities:

Lab session 6. PART II: Integration  
Lab session 7. Lab session exam (10%)

**Full-or-part-time:** 42h

Theory classes: 13h

Laboratory classes: 3h

Self study : 26h

## Linear algebra

### Description:

Matrices. Determinant. Rank of a matrix.  
Systems of linear equations. Gaussian elimination.  
Inverse matrix.  
Linear geometry: equation of a straight line and a plane; orthogonality and parallelism; distances.

### Specific objectives:

- Students will learn:
- to solve systems of linear equations.
  - to graphically represent the solution of a system of linear equations.

### Related activities:

Lab Session 4. Matrices  
Lab Session 5. Generic competence assessment

**Full-or-part-time:** 15h

Theory classes: 2h

Laboratory classes: 4h

Self study : 9h

## GRADING SYSTEM

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First partial exam: 35% (Test+Exercise)  
Second partial exam: 45% (Test+Exercises)  
Laboratory exam with MAPLE: 10%  
Generic competence:10%

Students can pass the course through the continuous assessment based on two exams (a first mid course exam and a second exam during the period fixed in the academic calendar of the school devoted to the final exams) and the delivery of laboratory assessments. Finally, as detailed in the academic normative of the EEBE, a reevaluation exam will take place (excluding the MAPLE Laboratory exam and the Generic Competence). To be able to do the reevaluation exam, the student has to attend to all the evaluation exams of the subject and its mark, N, for the part which can be reevaluated has to be such that 3.0

<https://eebe.upc.edu/ca/estudis/normatives-academiques/documents/eebe-normativa-avaluacio-i-permanencia-18-19-aprovat-je-2018-06-13.pdf>

An individual test will be performed in the assessment of the laboratory, during the last laboratory session, and another test will evaluate the generic competency. This course assesses the self-directed learning competency through individual tests during the development of one of the laboratory sessions. More precisely, the test will assess conics.

## EXAMINATION RULES.

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No writing paper, books, papers, manuscripts, or notes of any kind are allowed to be taken into an examination room. The use of calculators, cell phones, tablets, or any electronic device is not permitted during the exams.

## BIBLIOGRAPHY

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### Basic:

- Salas, Saturnino L.; Hille, Einar; Etgen, Garret J. Calculus : una y varias variables [on line]. 4a ed. Barcelona [etc.]: Reverté, 2011 [ Consultation : 16/04/2020]. Available on : [http://www.ingebook.com/ib/NPcd/IB\\_BooksVis?cod\\_primaria=1000187&codigo\\_libro=7715](http://www.ingebook.com/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=7715). ISBN 8429151567.
- Pozo, Francesc; Parés, Núria; Vidal, Yolanda. Matemáticas para la ingeniería [on line]. 2a ed. Madrid: García-Maroto Editores, 2019 [ Consultation : 02/10/2019]. Available on : [http://www.ingebook.com/ib/NPcd/IB\\_BooksVis?cod\\_primaria=1000187&codigo\\_libro=8434](http://www.ingebook.com/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=8434). ISBN 9788417969028.
- Lay, David C. Algebra lineal y sus aplicaciones. 4a ed. México [etc.]: Pearson Educación, 2012. ISBN 9786073213981.
- Thomas, George Brinton. Cálculo : una variable. 12ª ed. México, D.F: Addison Wesley Longman, 2010. ISBN 9786073201643.
- Rogawski, Jon. Calculus : single variable. 2nd ed. New York: W.H. Freeman and Company, cop. 2012. ISBN 9781429231831.
- Franco Brañas, José Ramón. Introducción al cálculo : problemas y ejercicios resueltos [on line]. Madrid [etc.]: Prentice Hall, cop. 2003 [ Consultation : 29/04/2020]. Available on : [http://www.ingebook.com/ib/NPcd/IB\\_BooksVis?cod\\_primaria=1000187&codigo\\_libro=1242](http://www.ingebook.com/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=1242). ISBN 9788483229620.

### Complementary:

- Gibergans Bàguena, Josep [et al.]. Matemáticas para la ingeniería con Maple. Barcelona: Edicions UPC, 2008. ISBN 9788483019672.

## RESOURCES

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### Hyperlink:

- Khan Academy. Resource

### Other resources:

Web page: <https://es.khanacademy.org>