UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

Course guide
320093 - C - Calculus

Last modified: 10/07/2023

Unit in charge: Terrassa School of Industrial, Aerospace and Audiovisual Engineering
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN AUDIOVISUAL SYSTEMS ENGINEERING (Syllabus 2009). (Compulsory subject).

Academic year: 2023 **ECTS Credits:** 6.0 **Languages:** Catalan, Spanish

LECTURER

Coordinating lecturer: JULIAN PFEIFFLE

Others: ROBERT VELASQUEZ

PRIOR SKILLS

It is considered very convenient to have assimilated the contents, procedures and norms and values corresponding to the mathematics courses in the syllabus of the different types of secondary education that give access to the studies of the degree.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
CE01-ESAUD. Ability to solve mathematical problems that may arise in engineering. Aptitude to apply knowledge of linear algebra, geometry, differential and integral calculus, differential and partial differential equations, numerical methods, numerical algorithms, statistics, and optimization. (Basic training module)

Basic:
CB1. That students have demonstrated possession and understanding of knowledge in a field of study that is based on general secondary education, and is typically found at a level that, while supported by advanced textbooks, also includes some aspects that involve knowledge from the forefront of their field of study.

TEACHING METHODOLOGY

- Sessions of presentation of theoretical content.
- Sessions of practical work.
- Self-guided study by the students, and completion of exercises.
- Preparation and realization of assessed activities individually and/or in groups.

In the sessions of presentation of content the teachers will introduce the theoretical basis of the subject, concepts, methods and results using appropriate examples to facilitate understanding.
The students, in an autonomous way, will study in order to assimilate these concepts and to solve the exercises proposed either manually or with the help of the computer.
Computer assisted learning tools will be promoted: the students will use a package of mathematical software to gain familiarity and with the aim that they use it as a tool for numerical, symbolic and graphical calculation.

LEARNING OBJECTIVES OF THE SUBJECT

The student will have to master the fundamental concepts of differential and integral calculus in one and several variables, in its analytical and numerical aspects. Sufficient knowledge of the manipulation and calculation techniques for handling the concepts that will appear during his or her further studies is expected to be acquired. Also the student is expected to look at computer tools for help, as he or she must become familiar with the appropiate mathematical software for the corresponding numerical, symbolic and graphical calculus applications, a skill that will prove useful in future modules. In this sense, the aim is to introduce him/her to numerical techniques for problem solving, in this case in the context of infinitesimal calculus, that play an ever increasing role in his/her studies.

STUDY LOAD

Type	Hours	Percentage
Hours medium group	30,0	20.00
Self study	90,0	60.00
Hours large group	30,0	20.00

Total learning time: 150 h

CONTENTS

Topic 1: DIFFERENTIAL CALCULUS OF ONE VARIABLE

Description:
1.1. Basic concepts. The set of real numbers, absolute value, inequalities. Domains.
1.2. Limits and continuity. Bolzano's theorem.
1.3. Inverse functions
1.4. Revision of elementary functions: polynomial, rational, powers, exponential, logarithmic, trigonometric, inverse trigonometric, hyperbolic, integer part.
1.5. The definition of derivative. Derivative function. Derivation rules.
1.6. Tangent line to the graph of a function at a point. Linear approximation.
1.7. Higher order derivatives. Lagrange's theorem. Taylor Polynomial. Taylor's Theorem. L'Hôpital's rule.
1.8. Classification of critical points of a differentiable function. Calculation of absolute and relative extrema. Optimization problems.
1.9. Graphical representation of functions in the plane.
1.10 Parametrized curves in the plane. Polar coordinates.
1.11 Vector functions in the plane. Vector fields in the plane.

Specific objectives:
The objective of the first unit is to consolidate the concepts and techniques acquired during previous studies of differential calculus in one variable, and to ensure all students acquire the necessary fluency and confidence in dealing with the functions and their graphs in the plane.
- It is essential that each student reflect on their weaknesses and to rectify them from the very beginning following the indications of the teacher and devoting the number hours of autonomous learning appropriate to each individual, that in this unit can be especially variable between students due to the various initial conditions possible.
- After reviewing, and studying in greater depth, the functions in one variable themselves, the unit ends with the study of parameterized curves and vector fields in the plane that are already known from earlier studies and that will facilitate the introduction of differential calculus in several variables that we study in the next unit
- The student must demonstrate the capability to:
- give sets of real numbers in set notation or using intervals, and display them graphically;
- determine the domains of elementary functions and graph them and calculate their values;
- apply elementary transformations to these functions, and calculate their inverses where possible;
- understand the concept of limit of a function at a point, from the left and right, and of limit if both exist and agree, and continuity of a function at a point, and the different types of intervals.
- know that if a function is continuous on a closed interval, then it must achieve maximum and minimum values at least once in this interval (Weierstrass theorem), and consequently know Bolzano's theorem that if a continuous function on a closed interval changes sign then it is zero at least once.
- understand the definition of derivative of a function at a point as the limit of the quotient of increments of the function near the point, if it exists, and interpretation of this as the gradient of the tangent line to the function at the point considered, and how to use it to give a linear approximation of the value of the function near the point using the tangent line.
- know how to determine when a function is differentiable at a point, and compute derivative functions from the functions that are differentiable in their domain, correctly using techniques of differentiation, including the product rule, quotient rule, chain rule and implicit differentiation.
- know the definition of higher order derivatives, Taylor polynomials around a point, the statement of Taylor's theorem, which gives the value of the function at a point near the point of expansion as sum of the value of the polynomial plus the value of the error term, and the form of the error term, and to determine an estimate for this term in simple cases.
- know how to apply the criteria for calculating relative extrema of functions, and how to determine and calculate absolute and relative extrema of continuous functions on closed intervals.
- know how to determine intervals of concavity and convexity.
- know how to resolve practical problems of optimization.
- know how to graph a function using information obtained from it (domain, intercept with the axes, zeros of the derivative, intervals of increase and decrease).
- know how to calculate the points of a parameterized plane curve and represent it graphically.
- know how to calculate the tangent vector of a parameterized plane curve and represent it graphically.
- know how to represent graphically vector fields in the plane.

Full-or-part-time: 27h
Theory classes: 6h
Practical classes: 6h

Self study : 15h

Topic 2: DIFFERENTIAL CALCULUS OF SEVERAL VARIABLES

Description:
2.1. Sets, domains and geometry in the space.
2.2. Maps with several inputs and several outputs; vector functions (one input and several outputs); scalar functions of several variables; vector fields (several inputs and several outputs).
2.3. Graphs. Level sets. Particular cases: level curves, level surfaces.
2.4. Limits and continuity for several variables functions.
2.5. Partial derivatives. Gradient vector. Directional derivatives. Total differential.
2.6 Chain rule. Curvilinear coordinates. Jacobians.
2.6. Tangent varieties (lines, planes). Linear approximations.
2.7. Higher order derivatives. Taylor polynomials. Quadric surfaces. Taylor Theorem. Parametrised surfaces.
2.8. Differential vector calculus: vector fields, differential operators.
2.9. Extremes calculus for several variables functions: Hessian rule. Conditioned extremes: Lagrange multipliers.
2.10. Practical applications: Neural networks, etc

Specific objectives:
The aim of unit 2 is to introduce the student to the differential calculus in several variables and to make him/her confortable in the use of it, emphasizing the contact with the repertoire of concepts and tools that he/she is bound to meet along his/her studies, although it will not be possible to get to the deeper knowledge of all of them because of lack of time.

We will begin where we finished the former unit, but on three dimensional space, by generalising the geometry of curves the was developed for the plane in the first unit.

One will need to become familiar with the concept of map from a set of points (given by several inputs) each of which is sent to an image point that can be given by several outputs as well, beginning with the easiest cases and generalising little by little. The interest will be stressed on the adquisition of new concepts and their understanding at an intuitive level, and the mastering of the basic examples and their manipulation not only on paper but also by means of symbolic, graphic and numerical software, as that will be paramount for the further development of the degree studies in our case.

The skills that the student should desirably acquire include:

-To be able to identify and interpret the level sets of functions, especially level curves. To interpret the gradient vector to a function at a point and its relation its level sets.
-To be able to compute and to intepret the value of the directional derivative of a function at a point in a given direction.
-To compute partial derivatives and to apply the chain rule, in particular to perform coordinate changes. To know the Schwarz lemma and to be able to compute higher order derivatives of several variables functions.
- To compute the equation of the tangent plane of a function of two variables at a given point and to use it to give the value of the linear approximation of the function at a point near to it. To generalise the understanding of this particular case of Taylor's theorem to higher orders (in practise order 2) an to use i to introduce usual geometric tools (quadrics, parametrised surfaces).
-To know the concept of vector field at a domain in space and the basic operators of differential vector calculus and the relations between them.
-To compute and classify the critical points of several variables functions, in particular two and thres variables, by means of the determinant of the Hessian matrix rule.
-To compute and classify the critical points of several variables functions with constraints in simple cases by the Langrange multipliers method.
-To identify and apply classical application cases of Stokes and divergence theorems in their differential form related to the studies in this degree.

Full-or-part-time: 48h
Theory classes: 10h
Practical classes: 8h

Self study : 30h

Topic 3: INTEGRAL CALCULUS

Description:
3.1. Indefinite integration. Calculus of primitives: elementary integrals; substitution integration; parts integration; rational integraion; trigonometric substitution.
3.2. Definite integration: concept of area; Riemann sums; definition of definite integral.
3.3 Relation between definite and indefinite integration: The Fundamental Theorem of Calculus. Barrow's rule.
3.4. Aplications of definite integration, including: work, calculus of areas, volumes, arc's length, revolution surfaces, moments.
3.5. Improper integrals: definite integrals with infinite integration limits, definite integratls with inifinite limit values, definite integrals with singulatrites, and combinations.
3.6. Iterated integrals and area in the plane. Double integration and volume. Variables change in the plane. Area of a surface. Triple integration and applications. Variable change in space.
3.7. Integral theorems in vector calculus. Stokes' theorem. Divergence (Gauss') Theorem.

Specific objectives:
The aim of unit 3 is to master integral calculus in one and several variables to the suitable level for the succesful completion of the degree programme, whre signal processing plays a very important role, and in particular for the Fourier Analysis and Differential Equations module. Emphasis will be put on the solid command of the basic skills in one variable, and the exposure to the new concepts in several variables so that the student would be able to face the technical tools that will need to deal with in the future.
In all cases work will include interaction between manual computation and use of mathetmatical software for the testing, support, plotting of the integration domains and of the graphs of the integrands. The Riemann sums based definition of the integra will allow as well to liase with the numerical analysis point of view that will be developed in a deeper way in the later unit.

The skills that is desirable to acquire include:

-To compute the almost immediate primitives with ease, adjusting the required parameters accordingly.
-To understand the relation between primitives computation and the most simple differential equations.
-To compute primitives of functions where easy substitutions are needed efficiently and reliably, using correctly the differential and change of variable.
-To apply correctly the parts integration formula when it is needed, repeatedly if needed so.
-To understand globally how the integration of rational functions work, and to be able to carry it out in several contexts.
-To compute definite integrals by applying Barrow's rule.
-To compute areas delimited by graphs of functions.
-To compute areas delimited by more general curves in the plane.
-To compute arc length of curves, volumes and surfaces (revolution) within the reach of the studied methods.
-To compute simple improper integrals of the kinds studied.
-To compute easy double integrals: to determine the integration domain, the integration limits, to apply Fubini's theorem.
-To compute easy triple integrals: to determine the integration domain, the integration limits, to apply Fubini's theorem.
-To apply the integral theorems of vector calculus to essential cases related to the degree content.

Full-or-part-time: 50h
Theory classes: 10h
Practical classes: 10h
Self study : 30h

Topic 4: NUMERICAL METHODS

Description:
4.1. Mathematical models in engineering. Exact and approximate solution to a problem. Exact and approximate value of a quantity. Absolute and relative error. Error sources. Estimation and bounds for errors. Propagation of errors.
4.2. Approximate resolution of nonlinear equations in one variable by iterative methods: bisection method, Newton method; convergence order; asymptotic error constant.
4.3. Concept of interpolation. Polynomial interpolation. Lagrange Interpolation. Newton's divided differences method. Interpolation error. Runge's phenomenon.
4.4. Functional approximation. Minimal squares approximation. Approximation error.
4.5. Numerical integration. Trapezium rule. Simpson's rule. Error formulas in numerical integration.

Specific objectives:
The aim of unit 4 is to direct the attention of the student to the importance of numerical methods in engineering. It is important the student understands the theorical concepts that comprise the first point of the table of contents of the unit in an automatic way, since without the assimilation of these the concrete calculations the concrete calculations carried out later will be of little use. In this sense, the concrete procedures detailed in the final 4 sections have the function of showing the student how the philosophy of numerical methods works in the context of content learnt in previous units, rather than of emphasising manual calculation. In this unit the role of mathematical software will be principal and the autonomous work of the student with this software in the context of the assigments will be the principal ingredient.
The desired skills to be reached are:

- to understand the notion of mathematical model.
- to understand the difference between exact and approximate solutions to a problem.
- to understand the difference between exact and approximate values of a quantity.
- to differentiate between the various types of error: relative and absolute, exact and approximate, and to be conscious of their sources, depending on the problem under consideration.
- to understand the concepts of estimation and bounding of errors.
- to know the principal mechanisms of error propagation.
- to know how to formulate a problem for approximate solution of a non-linear equation in one variable by an iterative method with specified conditions given on the permitted error. To be capable of applying it in concrete cases using the bisection method or Newton's method.
- to understand the concept of interpolation of a function for a set of points using one type of function. To know how to carry out the calculation in the particular case of polynomial functions by Lagrange's method and by Newton's divided differences method.
- to understand the concept of approximation of a function for a set of points using one type of function. To know how to carry out the calculation in the particular case of approximation by least squares. To know the behaviour of errors of approximation.
- to understand the concept of numerical integration to compute definite integrals, and its error formula. Know how to apply it in the cases of the trapezium rule and Simpson's rule.

Full-or-part-time: 25h
Theory classes: 4h
Practical classes: 6h
Self study : 15h

ACTIVITIES

LECTURE

Description:
Lectures during which the lecturer will introduce the theoretical bases of calculus and students will get familiar with them by means of observation and working through the first examples.

Material:
PDF files of the Beamer Presentations prepared by the lecturer available in the virtual campus.
Example sofware files prepared by the lecturer available in the virtual campus.
Class notes taken by the student.
Scientific calculator, pencil, eraser, ruler, lined paper, colours.
Chapters from the basic bibliography as indicated by the lecturer.

Full-or-part-time: 24h
Theory classes: 24h

TUTORIALS WITH IN CLASSROOM WITH BLACKBOARD

Description:
Tutorials where students under the guidance of the tutor will solve and share on paper and on the blackboard the exercises from the basic and advanced problem lists that the lecturer will have made available to them in good time for them to work independently beforehand. If convenient, they may use the help of the computer or other means. Work may be individual or in groups.

Material:
Basic problem class lists available in the virtual campus.
Additional problem class lists available in the virtual campus.
PDF files of the beamer presentations prepared by the teacher available on campus.
Example software files prepared by the lecturer available in the virtual campus
Problem sheets prepared by the individual student before the class session.
Class notes taken by the individual student.
Scientific calculator, pencil, eraser, ruler, graph paper, colours.
Chapters of the basic bibliography indicated by the teacher.

Full-or-part-time: 22h
Practical classes: 22h

PROBLEMS CLASS

Description:
Solving problems from the class lists, which may be basic or advanced, or other problems that the lecturer may specify after feedback and tutorial consultations.

Material:
Basic problem class lists available in the virtual campus.
Additional problem class lists available in the virtual campus.
PDF files of the beamer presentations prepared by the teacher available on campus.
Example software files prepared by the lecturer available in the virtual campus
Problem sheets prepared by the individual student before the class session.
Class notes taken by the individual student.
Scientific calculator, pencil, eraser, ruler, graph paper, colours.
Chapters of the basic bibliography indicated by the teacher.

Full-or-part-time: 22h
Self study: 22h

MATHEMATICAL SOFTWARE TUTORIALS

Description:
One computer lab tutorial session with mathematical software corresponding to each of the units will be held.

Specific objectives:
Those detailed in the content that are part of the corresponding activities.

Material:
All software is available in the computer rooms of the School.
The scripts and lists of problems are available on ATENEA and / or at the photocopying service.
Also, in the resources section of the course handbook there are links to free software of interest and other useful tools.

Delivery:
As a result of the session, everyone will have to submit through the virtual campus the tutorial logbook exercises performed with the relevant computer sofware. Each tutorial contributes 1.25% to the final grade.

Full-or-part-time: 12h
Practical classes: 8h
Self study: 4h

INDIVIDUAL HOMEWORK ASSIGNMENTS

Description:
Individual homework that the student will perform in an autonomous way to be delivered according to the specifications issued by the lecturer in the virtual campus.

Material:
Homework booklets available from the virtual campus.
PDF files of the Beamer Presentations prepared by the lecturer available in the virtual campus.
Example sofware files prepared by the lecturer available in the virtual campus.
Class notes taken by the student.
Scientific calculator, pencil, eraser, ruler, lined paper, colours.
Chapters from the basic bibliography as indicated by the lecturer.

Delivery:
Each unit will have associated an individual deliverable homework assignment which will account for 5% of the final grade.

Full-or-part-time: 50h
Self study: 50h

REVISION

Description:
Every student should revise in an organised way the theoretical and practical contents and the practical activities realised previously to the evaluation exams, and agree on an appointment with the lecturer in good time to clarify any doubts and to work on any possible weak points.

Material:
PDF files of the Beamer Presentations prepared by the lecturer available in the virtual campus.
Example sofware files prepared by the lecturer available in the virtual campus.
Class notes taken by the student.
Scientific calculator, pencil, eraser, ruler, lined paper, colours.
Chapters from the basic bibliography as indicated by the lecturer.

Full-or-part-time: 14h
Self study: 14h

EVALUATION EXAMS

Material:
Exam booklet and / or other materials as specified by the lecturers.

Full-or-part-time: 6h
Theory classes: 6h

GRADING SYSTEM

There will be two partial exams with mandatory assistance. The final grade decomposes as follows:
- Part 1: 45% (first partial exam: 40% + additional coursework 5%)
- Part 2: 55% (second partial exam: 45% + additional coursework 10%)
An insufficient result in the first exam can be compensated via a written exam to be taken on the same day as the second partial exam. This exam may be taken by all students with grades strictly less than 5 in the first written exam. The compensatory exam will be graded from 0 to 5, and this new grade will substitute the old one if it is greater.

For those students who meet the requirements and submit to the reevaluation examination, the grade of the reevaluation exam will replace the grades of all the on-site written evaluation acts.
If the final grade after reevaluation is lower than 5.0, it will replace the initial one only if it is higher. If the final grade after reevaluation is greater or equal to 5.0, the final grade of the subject will be pass 5.0.

BIBLIOGRAPHY

Basic:
- Gibergans Báguena, J. [et al.]. Matemáticas para la ingeniería con Maple. Barcelona: Edicions UPC, 2008. ISBN 9788483019672.
- Strang, Gilbert. Calculus. Wellesley: Wellesley-Cambridge Press, 2017. ISBN 9780980232752.
- Chapra, Steven C.; Canale, Raymond P. Métodos numéricos para ingenieros [on line]. 7ª ed. México: McGraw-Hill, 2015 [Consultation: 08/03/2023]. Available on: https://www-ingebook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=8100. ISBN 9781456267346.
- Larson, Ron; Edwards, Bruce H. Cálculo. Vol. 1, De una variable [on line]. 9a ed. México [etc.]: McGraw-Hill, 2010 [Consultation: 14/06/2022]. Available on: https://www-ingebook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=5686. ISBN 9786071502735.
- Larson, Ron; Edwards, Bruce H. Cálculo. Vol. 2, De varias variables. 9a ed. México [etc.]: McGraw-Hill, 2010. ISBN 9789701071342.

Complementary:
- Burden R.L.; Faires J.D. Análisis numérico. 6ª ed. México D.F: International Thomson, 1998. ISBN 968-7529-46-6.
- Yasskin, Philip B. [et al.]. CalcLabs with Maple: single variable calculus. Belmont: Brooks/Cole, Cenegage Learning, cop. 2010. ISBN 9780495560623.
- Puig i Sadurní, J. Taller de matemàtiques: pràctiques en Matlab/Octave, amb un apèndix en Python [on line]. Barcelona: Iniciativa Digital Politècnica, 2011 [Consultation: 14/05/2020]. Available on: http://hdl.handle.net/2099.3/36550. ISBN 9788476536568.
- Quarteroni, A.; Saleri, F. Cálculo científico con MATLAB y Octave [on line]. Milano: Springer, 2006 [Consultation: 14/06/2022]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-88-470-0504-4. ISBN 9788847005037.
- Schmidt Hansen, J. GNU Octave: beginner's guide: become a proficient Octave user by learning this high-level scientific numerical tool from the ground up [on line]. Birmingham: Packt Publishing, cop. 2011 [Consultation: 03/05/2022]. Available on: https://web-p-ebscohost-com.recursos.biblioteca.upc.edu/ehost/ebookviewer/ebook?sid=46c177c7-8d61-4ea8-8490-0d8b1750ee42%40redis&vid=0&format=EB. ISBN 9781849513326.
- Arias, Irene [et al.]. Cálculo avanzado para ingeniería: teoría, problemas resueltos y aplicaciones [on line]. Barcelona: Edicions UPC, 2008 [Consultation: 14/05/2020]. Available on: http://hdl.handle.net/2099.3/36849. ISBN 9788483017609.
- Perelló, Carles. Càlcul infinitesimal amb mètodes numèrics i aplicacions. Barcelona: Enciclopèdia Catalana, 1994. ISBN 8477395187.
- Leseduarte, M. C.; Llongueras, M. D.; Magaña, A. Càlcul I: teoria i exercicis [on line]. Barcelona: Iniciativa Digital Politècnica, 2011 [Consultation: 14/05/2020]. Available on: http://hdl.handle.net/2099.3/36651. ISBN 9788476537312.
- Salas, Saturnino L.; Hille, Einar; Etgen, Garret J. Calculus: una y varias variables [on line]. 4ª ed. Barcelona [etc.]: Reverté, 2002 [Consultation: 14/06/2022]. Available on:

https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=5635414. ISBN 8429151567.
- Chavarriga, Javier; García, I. A.; Giné, Jaume. Manual de métodos numéricos. Lleida: Universitat de Lleida, 1999. ISBN 8484099989.

RESOURCES

Computer material:
- MAPLE. Mathematical software available for use at the computing labs.
- Matlab. Resource

Hyperlink:
- http://sage.math.washington.edu/home/wdj/teaching/calc1-sage/. Differential Calculus with SAGE
- http://www.gnu.org/software/octave/. Resource